The current research report is part of a research project that is implemented within the framework of the Action «Supporting Postdoctoral Researchers» of the Operational Program "Education and Lifelong Learning" (Action's Beneficiary: General Secretariat for Research and Technology), and is co-financed by the European Social Fund (ESF) and the Greek State.

Κωδικός Πρότασης & Ακρωνύμιο Πρότασης: **SH4_3510 "EDARCPADSS"**

Τίτλος Πρότασης: **Epistemological and Didactical Aspects Related to the Concept of Periodicity Across Different School Subjects** (Επιστημολογικές και διδακτικές απόψεις σχετικές με την έννοια της περιοδικότητας σε διαφορετικά σχολικά μαθήματα).

Φορέας Υποδοχής: **ΑΝΩΤΑΤΗ ΣΧΟΛΗ ΠΑΙΔΑΓΩΓΙΚΗΣ ΚΑΙ ΤΕΧΝΟΛΟΓΙΚΗΣ ΕΚΠΑΙΔΕΥΣΗΣ (Α.Σ.ΠΑΙ.Τ.Ε.)**

Ερευνητική μονάδα φορέα υποδοχής: **ΓΕΝΙΚΟ ΤΜΗΜΑ ΠΑΙΔΑΓΩΓΙΚΩΝ ΜΑΘΗΜΑΤΩΝ (ΓΕΤΠΜΑ)**

Στοιχεία Μεταδιδάκτορος Ερευνήτριας: **ΤΡΙΑΝΤΑΦΥΛΛΟΥ- ΚΑΛΛΙΩΡΑ ΧΡΥΣΑΥΓΗ**

Στοιχεία Επιστημονικής Υπευθύνου: **ΣΠΗΛΙΩΤΟΠΟΥΛΟΥ ΒΑΣΙΛΙΚΗ, Καθηγήτρια**

DELIVERABLE-PHASE 4 (ΠΑΡΑΔΟΤΕΟ 4ης ΦΑΣΗΣ)

FINAL REPORT of the RESEARCH PROJECT

CONTENTS

Ch. 1

INTRODUCTION

The overall aim of the research project

Our research project intended to take a close look at pedagogical practices adopted in mathematics and physics classrooms in Greek Lower and Upper secondary schools on topics that are related to periodicity. Even though periodicity is central in a variety of disciplines, an extensive search of the literature shows that there are only a limited number of studies that focus on its understanding.

Periodicity is a concept present in the development of scientific thought. Through history, periodicity was used as an argument that qualifies a certain repetitive behaviour and from there, under a more analytic use, was a property for functions that modelled oscillatory movements. Starting with pattern observation, human beings are capable of abstracting this property in order to generate scientific knowledge. Examples of its use while developing scientific knowledge are numerous: Pannekoek (1961) identifies the systematic observation of celestial bodies' periodic behaviour as the origin of astronomy as a scientific activity; Whitehead (1983) points out periodicity as the property which favours an analysis of the analogies between different physical phenomena. It turns out to be a property of different kinds of objects that begins in the everyday individual experiences (the year seasons, night and day), it enters into school's mathematics from the very beginning (periodic decimal numbers) and goes through several school's disciplines (phenomena in physics, functions as in calculus) all of which form part of the students' scientific culture.

Even though periodicity is central in a variety of disciplines, an extensive search of the literature shows that there are only a limited number of studies that focus on its understanding. These studies conclude that most students' conceived image of periodicity is based on time-dependent variations (Shama, 1998), while usually they consider any repetition as being periodical (Buendia & Cordero, 2005). Dreyfus and Eisenberg (1980) state that textbook authors appear to be well aware of the technical difficulties often arisen while proving that a function is periodical. In addition, as Van Dormolen and Zaslavsky (2003) argue, the periodical property changes according to what is considered periodical in a particular field. Buendia and Cordero (2005) argue that notion's current treatment in school due to different perceptions of and practices in science and mathematics instruction limits its recognition.

The three research phases

In this research study we take the position that understanding the notion of periodicity and its properties involves creating a coherent framework where ideas and educational practices in different school subjects are meaningful at an individual level. Furthermore, understanding of periodicity is realized through specific situations where it takes its meaning (Radford, 2013).

To meet the aims of our inquiry, we designed three different research phases. In the ***first phase*** of our project we analyzed Greek textbooks taken from the subjects of science (physics, astronomy, applied technologies) and mathematics focusing on the reasoning practices adopted in the textbooks; we have analyzed 110 thematic units; 214 visual representations and 162 proposed exercises on 12 textbooks.

In the ***second phase***, our main interest was how undergraduate students in the first and second year of their studies perceive periodic motions and their graphical representations. We contacted three research activities. 288 undergraduate students (230 from the technological education and 58 from the technological education) from 4 Technological departments and 2 University departments participated. The participants were in scientific direction fields who passed at a national exam in order to attend their undergraduate studies. Part of their exams was thematic units on periodicity (e.g. mechanical and electrical oscillations). Besides, during their undergraduate studies all students in scientific direction fields encounter aspects of periodicity (usually in their first year Calculus and Fourier analysis courses). Fourier analysis is a prerequisite course for studying signal processing in the fields of Informatics and Electronics. Thus, for all the participants, periodicity is considered as an important scientific notion not only for their academic studies, but for their professional life as well. Our emphasis was mostly in undergraduate engineering students who study in Technological Institutes because (i) in Greece holds lower prestige than Universities, (ii) is an area that is under investigated, and (iii) the notion of periodicity is central in engineering studies.

Finally, in the ***third phase*** we focused on secondary teachers' practices when they teach topics relative to periodicity in their classes. 55 teachers participated in two research activities.

Summative results taken from the first, second and third reports

In the first research we focused on the reasoning practices adopted in Greek mathematics and science textbooks when new knowledge on periodicity is developed. Reasoning has been investigated through the logical act created by a part of the text of a thematic unit. We realized that the argumentation in a thematic unit is produced by the sequence of the ***Modes of reasoning (MsoR)*** that the author develops in the text when organizing and presenting the new knowledge, The main categories of modes of reasoning identified were (a) the *empirical,* the *logical-empirical*, the *nomological,* and the *mathematical*. Furthermore, our analysis indicated that Physics is the richest subject where different aspects of periodicity are introduced explicitly and developed. The exploration of the function of VRs in all textbooks reveal tools, practices and rules used by the different communities and show the nature of activity taking place. In this way ***the sinusoidal graph*** dominates all school activities. Emphasis on abstracted aspects of periodicity is apparent in mathematics where almost all visual representations are context free (Triantafillou & Spiliotopoulou, 2014). Physics is starting with the presentation of natural and everyday phenomena and follow a path up to semi-abstracted forms of knowledge (showing time-dependent images). The other two subjects follow their historical goals (examples of Natural life are met mostly in Astronomy while images of scientific devices are mostly met in Applied Technologies). Finally, our analysis indicated that there is divergence in the demand in the proposed exercises analyzed in the mathematics and science subjects while most of all the proposed exercises are context free while VRs are almost absent in both subjects.

The results in the second phase indicate that the vast majority of students easily identified the periodical property in periodic graphs. It is interesting though that a graph, which exhibits a fluctuation (looks like the sinusoidal curve but with decreasing amplitude) seemed to confuse students a lot since almost seven out of ten considered that this represents a periodic motion. Conceptualizing proportional relations of the quantities E (in Voltage) -υ (in m/s^2) on the formula E=Blυsina is more difficult than conceptualizing proportional relations of the quantities F-x on the formula F=kx. This is explained since the second formula is the typical formula of examining linear relations in mathematics, while the first one is not. Besides, students rarely make connections between different graphical representations of the same periodic phenomenon. Although argumentation and reasoning seems to be a non-familiar practice for students they exhibit strong willingness to assign meaning to abstract mathematical entities (Triantafillou, Spiliotopoulou & Potari, 2014). Nomological MsoR as the definition of periodic functions as appears in mathematics texts, seems to be not a familiar tool for arguing even in the case of students who are studying mathematics. Undergraduate engineering students - prospective teachers in Greek secondary vocational schools – transformed a text on a thematic unit that was referring to the periodical character of car suspension into an explanatory teaching unit exhibit three levels of awareness: the superficial, the partial and the substantial explanatory. In the latter case students' explanatory text was argumentative while in the other cases was mostly descriptive.

The results of the third phase indicate that the fundamental image of the notion in teachers' pedagogical practices in mathematics and science is the sinusoidal function. All teachers mention a lot of examples used in their lesson when teaching aspects of periodicity and they suggested that making connections to everyday life periodic phenomena could help students to develop a unified view of periodicity. But our analysis indicated that teachers are not consciously use every day phenomena as generic examples or in order to make a general claim but only to stimulate their students' attention. All the educators were critical of the textbooks use in their classroom practice so they prefer to modify parts of the new knowledge organization provided in each thematic unit. The modifications mentioned by the teachers could be omitting or enriching the modes of reasoning presented in their texts. These modifications could result in changes in the argumentation developed in each thematic unit and hence influence students' conceptualization. Finally, co-operation among teachers from different subjects seems to be a non-preferable practice by almost all of them. Hence, instead of identifying differences in teacher's pedagogical practices when they teach aspects of the notion of periodicity we identified common reasoning attitudes.

OUR AIM in the FINAL REPORT

In the final report our focus is on the reasoning practices a) developed in school texts, b) adopted by the students in the process of making sense of textual and visual elements on periodicity and c) implemented by secondary teachers when teaching periodicity. Our further aim is to detect links between argumentation and conceptualization or in other words to find out how the reasoning practices are interrelated with conceptualizing aspects of the notion of periodicity.

In order to implement our plan in some cases we repeat our initial analysis under our developed and established framework on modes of reasoning (Triantafillou, Spiliotoulou & Potari, 2015). This framework provide us a set of filters through which we can systematically examine a) science and mathematics textbooks in terms of how knowledge is presented and argumentation is unfolded not only through the texts of units, but also through textbooks' proposed exercises, b) the students' responses in selected tasks on periodicity and c) teachers' practices in terms of teaching periodicity and the use of textbooks' argumentation.

We divide the final report in two parts.

PART I: The Nature of argumentation adopted in Greek School Mathematics and Physics Texts on Periodicity.

In the first part, we focus on the nature of argumentation adopted in school texts in the 'content presentation sections' and in the reasoning practices demanded by the students when they are engaged in the 'textbook proposed exercises'. Our discussion in the final report will take place only about the subjects of mathematics and physics, since our initial analysis indicates that these are the main school subjects that introduce students in Greek lower secondary and upper secondary general and vocational schools to the notion of periodicity.

PART II: We divide this part of our study in ***PART IIa*** and ***PART IIb.***

In ***PART IIa,*** we focus on undergraduate students' meaning making of textual and visual elements of school texts on periodicity. We analyze their justifications on certain tasks related to aspects of the notion. The title of this part of our research is ***"Documentation of undergraduate students' thinking and reasoning on periodicity".***

In ***PART IIb,*** our focus is on secondary mathematics and science teachers' pedagogical practices when teaching specific thematic units on periodicity. The title of this part of our study is: ***"Teachers' pedagogical tools when teaching periodicity".*** Particularly, we are seeking to discover how teachers of the two disciplines use texts' inherit logic when teaching aspects of periodicity and how they institutionalize their students' knowledge accordingly.

The findings of our project reveal practices adopted in the two communities of mathematics and physics and can help us to build a wider perspective of how reasoning is related to conceptualization in the case of periodicity. Moreover, issues like the transfer of content knowledge and its relation to the transfer of reasoning skills in the two communities are elaborated. Pedagogical implications of our findings can be relevant to the teaching and learning of the related school subjects and the development of innovative curricular materials that help students develop their reasoning and argumentation in order to develop a robust understanding.

Ch. 2

PART I

“The nature of argumentation in Greek school mathematics and physics texts on periodicity”

INTRODUCTION

Reasoning, the human capacity to make sense of the world, has long been the goal of science and mathematics. Despite the obvious differences in the two subjects’ themes, Lakatos (1976) pointed out the strong parallels between mathematical and scientific reasoning discourse. From a pedagogical point of view, Vygotsky and Piazet established the value and the role of argumentation in students' thinking. For example, in *Judgment and reasoning in the child* Piazet (1928) described the development of logical thinking as the result of children's confrontation with other points of view and the subsequent need to justify their own view points. Vygotsky (1978) by considering the important point of social interaction emphasizes that understanding emerges through differences and arguments. In this direction, a common learning objective in mathematics and science education is to help students gain understanding of how scientific/mathematical claims can be proved or disproved (Oehrtman & Lawson, 2008). Argumentation as the act of forming reasons, making inductions, drawing conclusions, and applying them to the case under discussion is met in students' responses and in teachers' practices as well.

One didactical resource of appropriate reasoning practices is the school texts (Nicol & Crespo, 2006; Koponen & Nousiainen, 2012). In secondary school education in most countries (Greece included) textbooks are the main source of potential learning. Not only textbooks are the expressions of the *intended curriculum* (the goals and objectives intended for learning at a national level), but teachers also use them as the main (and maybe the only) resource to assign homework to their students. In spite of the pervasive presence of textbooks in schooling and educative practices, few research studies have focused on textbooks analysis in relation to their content (e.g. Stinner, 1992; Mesa, 2004). Moreover, there is no research in textbook analysis on a particular notion that crosses different educational courses. In this study we analyze the inherent logic of a concept presentation in mathematics and science school texts and the problems presented to students in specific topics related to the notion of periodicity.

THEORETICAL BACKGROUND

Vygotsky (1978) distinguished two types of concepts. The every day or spontaneous or intuitive concepts arising from students' experiences and the scientific or theoretical or formalized concepts or the cultural tools that have been elaborated and refined in a school or academic community. Mature knowledge is achieved with the merging of every day and scientific concepts and not by replacing the former by the latter ones. But what about the notion of periodicity that is very close to all students' experiences and at the same time a fundamental scientific concept with different images in mathematics and science?

Physics and math teachers in recent days are very focused on improvements in teaching their discipline. Their purpose is to ensure that students understand the concepts relevant to the field. In math, the emphasis is on improving skills and simplifying conceptual development without explicit attention to its connections to physics and engineering. On the other side, the selected physics curriculum reduces the complexity of the underlying mathematical content by keeping it at the level of algebra or trigonometry. Bingolbali and Monaghan (2008) by comparing the math content between the calculus and the physics textbooks argued that there is evidence that each holds differing epistemological and paradigmatic commitments, which are not incompatible but whose connections are not made explicit to the students. So, how school texts help students to make the appropriate connections about a common concept as the notion of periodicity?

In this part of the report, we restrict out attention to the argumentation practices developed in school texts.

The Meaning of Argumentation

van Eemeren and Grootendorst (2004) defined argumentation as a *verbal* (uses oral and written language), *social* (involves two or more people), and *rational* (intellectual) activity aimed at defending a standpoint. Argumentation has three generally recognized forms: *analytical*, which is grounded in the theory of logic and proceeds inductively or deductively from a set of premises to a conclusion; *dialectical*, which occurs during discussion or debate; and *rhetorical,* which is employed to persuade an audience. In our study, we consider that the argumentation developed by school textbook authors is a combination of analytical and rhetorical elements, links, and moves employed to persuade the readers (e.g., students or educators). If these two forms are successful, the dialectical form of argumentation occurs and can be documented. We also study argumentation beyond its verbal component to its visual components, which are expressed through representations (e.g., drawings, pictures, graphs, charts, tables, models, and images).

In most studies, the analysis of science and mathematics argumentation practices is based on Toulmin's (2003) framework with the following elements: claims, data, warrants, backings, qualifiers, and rebuttals. Claims are statements that advance a position taken. Data involves observations, facts, measurements, etc. that can be used as evidence to prove or support the claim. Warrants are the logical connections between data, backings, evidence, and claims that indicate support for the claim or the rebuttal of a counterclaim. Backings support the validity of the warrants. Qualifiers refer to the degree of strength and certainty in one's own argument, while rebuttals challenge any element of arguments put forth by others. Although logic is seen as an academic discipline that presents decontextualized rules for relating premises to conclusions, arguing is a human practice that is situated in specific social settings (Toulmin, 2003). From this perspective, textbook argumentation can be seen as a negotiated sociocultural act that takes place within a group of people (e.g., authors, students, and educators) in a school/learning community.

A central problem area in the analysis and evaluation of argumentative discourse is the analysis of "argumentation structures" (van Eemeren & Grootendorst, 2004, p. 2). The "single argumentation consists of a single reason for or against a standpoint, while in argumentation with complex structures, several reasons are put forward for or against a standpoint" (p. 4). We consider that the argumentation developed by the author in a school textbook is usually characterized as having complex argumentation structures because different forms of reasoning are used to persuade the readers. In the present study, argumentation is taken as the sequence of the forms of reasoning that the author develops in a text when organizing and presenting the new knowledge.

Argumentation and Reasoning in Mathematics and Science Textbooks

Numerous research studies have been conducted on the analysis of reasoning, explanations, and proving or supporting ideas in school mathematics and science textbooks. The focus related to argumentation in mathematics textbooks has been on the type of reasoning and on the nature of the mathematical activity that is promoted. van Dormolen (1986) identified *theoretical* (theorems, definitions, axioms), *algorithmic* (explicit methods or how to do a specific operation or procedure), and *logical* (statements about the way one should work using the theory) dimensions in analyzing the mathematics evident in textbooks. Stylianides (2009) presented an analytic/methodological approach for the examination of the opportunities designed in mathematics textbooks for students to engage in reasoning and proving activities in which he identified *non-proof* and *proof arguments*. Non-proof arguments involve the cases of empirical arguments and rationales that capture valid arguments for or against a mathematical claim; proof arguments involve generic examples (e.g., particular case seen as representative of the general case) and demonstrations (e.g., connected sequence of assertions based on accepted truths such as axioms, theorems, definitions, and statements). Stacey and Vincent (2009) analyzed the nature of data and warrants presented in mathematical textbook explanations and identified the following modes or categories of reasoning: *deductive* (using a model or a specific or general case), *empirical* (experimental demonstration or concordance of a rule with a model), *metaphorical*, and *appeal to authority*.

Similarly, analysis of the argumentation in science textbooks focuses on the forms of reasoning and on the scientific dimensions emphasized. Fahnestock and Secor (1988) included *definitions* and *generalizations* as two major elements that are evident in science textbooks. Stinner (1992) classified the knowledge provided in science textbooks in two planes: the *logical plane,* which includes the finished products of science such as laws, principles, models, theories, and the mathematical and algorithmic procedures establishing them; and the *evidential plane,* which includes the experimental, intuitive, and experiential connections that support the logical plane. Moreover, Mahidi (2013) found that the knowledge organization in university-level physics textbooks on specific topics used inductive-like and deductive-like structures. Overall, the literature review indicates that there are common grounds in reasoning in these two school subjects. For example, empirical types of reasoning and the more formal types of reasoning (including definitions, mathematical algorithmic procedures, and generalizations in the form of mathematical proofs or deductions that are based on general cases) are encountered in both mathematics and science school practices.

Several studies on mathematics and science textbooks focus exclusively on the nature and extent of opportunities for students to engage in reasoning practices. Dolev and Even (2013) reported opportunities for justifications and reasoning they found in Grade 7 mathematics algebra and geometry textbooks. Stylianides (2008) focused on how proof is promoted in mathematics curriculum materials in a reform-based mathematics curriculum. Pegg and Karuku (2012) analyzed reasoning practices in junior high school chemistry textbooks, and McComas (2003) analyzed U.S. secondary school biology texts with respect to how the concepts of *law* and *theory* were defined and applied. These findings indicate that texts rarely require students to evaluate or apply scientific claims and that further research is needed in the field of students' understanding and reasoning practices adopted in school textbooks.

Finally, Croarke (1996) believes that expanding definitions of argumentation beyond the verbal component is necessary while Lemke (1998) argues that scientists use a semiotic combination of text, mathematical expressions and images (e.g. graphs, photos) in order to reason. In this case, readers must interpret the verbal and the visual components of the document to comprehend authors' arguments. Representational practices have a central role in science and mathematics school communities (Arcavi, 2003; Latour, 1987). In the context of physics Clark and Mayer (2008) argues that adding graphics to text can improve learning while visual representations are considered to be legitimate components of scientific arguments and explanations. Representational practices have a central role in science and mathematics school communities (Arcavi, 2003; Latour, 1987). Visualization, as a method of 'seeing the unseen' in images (Arcavi, 2003, p. 216), is no longer related to the illustrative purposes but is also being recognized as a key component of reasoning deeply engaging with the conceptual and not the merely perceptual aspect of knowledge (ibid.). In this direction, Biehler (2005) considers that the representations available for working with are essential elements constitutive of the meaning of any scientific concept. Visual means of communication are particularly helpful in introducing abstract concepts in science and mathematics. Fond, Godino and D' Amore (2007) argue that to speak about visual representations (graphs, tables, photos etc.) is equivalent to speaking about knowledge, meaning, comprehension and modelling since these notions make up one of the central nuclei in the disciplines of mathematics and science.

Periodicity is a concept related not only to periodical phenomena of everyday life and natural world, but to abstract mathematical notions which model them, as well. Periodical phenomena are translated through series of visual images in school textbooks that are at once more inclusive but also more distant from the direct experience with the phenomena. Researchers argue that students facing difficulties in handling and integrating the conceptual (e.g., periodic graphs) and the perceptual (e.g., the periodic motion of a pendulum) aspects of periodicity (Buendia & Cordero, 2005). The question of how textbooks support this difficult integration is an important and open issue (Dreyfous & Eiseberg, 1980). Scientists use various practices associated with visual tools. Moreover, scientists construct inscriptions in order to express ideas for a given task and use inscriptions to explain phenomena, make predictions, and as forms of communication (Kozma, Chin, Russell, & Marx, 2000). Some of these inscriptions are also included in school textbooks and the way they function inside the text could reveal the pedagogical practices adopted in the school communities (Pozzer-Adernghi & Roth, 2004).

Adopting an activity theory perspective, visual representations are considered as 'elements' (i.e. the basic building blocks) of activity (Roth & Lee, 2007). The way these elements are used in the fields of science and mathematics and contribute to collective knowledge on periodicity is important for students' learning. This means that the presence of the concept of periodicity in the school curriculum cannot be understood or analyzed, without reviewing the pedagogical practices adopted in the textbooks of these two communities. In both mathematics and science pedagogical practices when teaching aspects of periodicity are images of instances (or aspects or properties or models) of the notion. These representations in a school text are expressed either visually (e.g. pictures, diagrams or maps) or symbolically (e.g. equations or formulae). The role of images of a common notion in different teaching practices is under investigated so far. We consider that the representations of the notion of periodicity are cultural resources who acted as bearers of distributed intelligence (Pea, 1993) that they carry, in a compressed way, socio-historical experiences of cognitive activity and artistic and scientific standards of inquiry (Lektorsky, 1984). These ubiquitous mediating structures both organize and constrain educators' teaching practice and provide to students a specific conceptually structured space to think (Radford, 2013).

Viewing learning as participation in a well-defined practice, tools and artifacts and their use in this practice are considered as primary object of study (Sfard & McClain, 2002). In this direction, different studies have focused on the analysis of visual representations. Levin (1981) has identified five functions concerning how pictures serve in text processing - four conventional functions (decorational, representational, organizational, interpretational) and one more unconventional one (transformational).

According to Pozzer-Adernghi and Roth (2004) all representations lie along a continuum from least to more abstract depending on the amount of contextual detail that they carry in the background of the central object. This continuum starts with photographs (as images full of gratuitous detail) and naturalistic drawings, continuous with maps, diagrams, graphs and tables, and ends with mathematical equations (as the most abstract images).

Different views and analytical approaches on the visual codes have been used in the science context. Kress and van Leeuwen by following Haliday's seminal work on Systematic Functional Linguistics define images as representing ideas about the world (the ideational function); develop a relationship between illustrator and audience (the interpersonal function); and provide cohesive links (the textual function) (Haliday, 1985; Kress and van Leeuwen, 2006). Pozzer-Adernghi & Roth (2004) acknowledge that photos and their captions are playing a fundamental role in main text reasoning. In this direction they identified the following functions of photographs and their captions in interpreting the main text in Biology textbooks: decorative (there is no caption and there is no reference from the main text to the photograph); illustrative (include a caption that describes what the reader is to see in the photograph but the caption does not provide additional information to the main text); explanatory (captions provide an explanation of or a classification of what is represented in the photographs); and complementary (captions add new information about the subject matter treated in the main text). They conclude that these differences will influence readers' interpretations of the photographs and change their role in the text.

Purpose of PART I of our study and research questions

We adopt the position that textbooks aim to introduce their readers to the conceptual aspects of scientific and mathematical knowledge and persuade them of their value. This implies that, inside the text, the deployment of argumentation and conceptualization is inevitable, while learning is viewed as occurring through the dialectical relationship between these two channels of thought.

In our study, we consider that the argumentation developed by an author in a school textbook is a combination of analytical and rhetorical arguments (employed to persuade the reader who in our case could be a student, or a reader or an educator). If these two forms are successful, dialectical form of argumentation could also occur. Furthermore, the function of the visual representations in relation to the reasoning developed in the science or mathematics text is investigated. We take the view that the VRs' genre and the co-deployment of VR and mode of reasoning influence the argumentation developed in a school text and consequently influence the practices adopted in the Mathematics and Physics school communities.

By restricting our attention to thematic units related to the notion of periodicity in Greek mathematics and physics textbooks, we address the following research questions:

- How is argumentation structured and developed when employed in the texts on periodicity?
- What is the role of visual representations in school texts' reasoning?
- In what respect are argumentation practices similar or different in school mathematics and physics textbooks?
- How is argumentation unfolded and co-deployed with conceptual aspects of periodicity while reading a thematic unit?
- What are the conceptions of periodicity that may be stimulated by the solutions to exercises and problems in the given sample?

METHODOLOGY

We used a grounded theory research approach (Corbin & Strauss, 2007) partly in response to an increasing awareness of the limitations of applying *a priori* deductive theories to human transactions embedded in a social or an educational context and partly in response to the lack of an existing scheme of categories broad enough to allow us to study how periodicity is presented and argued across mathematics and science texts. Grounded theories are situated, not only in "the data," but also in the context in which the data were collected and may be considered idiographic theories. Quality criteria for idiographic theories of action emphasize transferability or adaptation to different contexts (Gasson, 2003). Inductive content analysis (Mayring, 2000) has been applied on specific thematic units in mathematics and physics texts that present periodicity. We were looking for categories emerging in terms of existing forms of reasoning and argumentation inherent in these texts with the aim of producing a common coding system of categories.

The context

The Greek school system is organized in terms of primary school (ages 7-12), lower secondary school (ages 13-15) and upper secondary school (ages 16-18). Mathematics and physics are compulsory subjects in the last two years of primary, and all years of lower and upper secondary education, and cover a considerable part of the weekly teaching schedule. The Educational Policy Institute of the Ministry of Education establishes a national curriculum for each school subject and grade level that are accompanied by prescribed textbooks. The textbooks are mandatory for all public and private schools and are normally distributed free of charge to all students and teachers in the public education sector. This makes the textbooks the central resource for deciding what topics are studied and how they will be studied.

The domain of our analysis

The texts analyzed are taken from eight Greek textbooks (four Mathematics and four Physics) used in Greek Lower Secondary and Upper Secondary General Schools. Particularly, one physics and one mathematics textbook taught in Lower secondary school in 3rd grade (ages 14-15), while two physics and two mathematics textbooks are taught in 2nd grade (ages 16-17) and one physics and one mathematics in 3rd grade (ages 17-18) of Upper secondary school.

Mathematics and physics were selected as the concept of periodicity is presented to a larger extent in comparison to other subjects that also involve periodicity, such as astronomy or engineering books or chemistry. We restricted our analysis to topics that are related to periodicity in the selected textbooks. Specifically, in mathematics, the selected sections were related to trigonometry and periodic functions and in physics they were related to periodic motions such as electrical and mechanical oscillations. In this part we analyze only the 'content presentation' sections of each thematic unit

In order to implement our analytic plan, we divided the selected 'content presentation' sections into thematic units by restricting analysis to all the parts which aim at presenting new mathematical and scientific knowledge. In this part of our analysis we excluded exercises and problems aimed to be solved by the students (usually placed at the end of the units) and historical notes (placed as separate texts and usually printed in different colour of paper) that have an informative character of the historical dimension of aspects of periodicity in each field. We name these textbook sections as 'content presentation sections'.

Our thematic unit is conceived as a part of the selected textbook section; it has a beginning and an end; and has a relative independence in its content i.e. we can identify and distinguish it from its neighbouring thematic units. Each thematic unit is characterized by its thematic content (e.g. "Define periodic functions" or "Define periodic motions") or it has an easily identifiable central idea. One thematic unit several times coincides with a textbook unit as it is defined by the author. But in some cases we have to split the textbook unit in more than one thematic unit when a change in its thematic content is identified**.** We analyzed a total of 72 thematic units—29 units from Mathematics and 43 units from Physics. Each thematic unit has a complete conceptual meaning, which is supported and validated by a series of logical acts and sequences of reasoning. It, also, has a logical structure, which is viewed in our study as the argumentation developed in each thematic unit. Reasoning and argumentation are the main focus of our analysis**.**

Subsequently, in order to characterize students' practices related to the notion of periodicity we analyzed exercises and problems aimed to be solved by the students, usually placed at the end of the units and we name these units 'proposed exercises sections'.

The sample

In the Table I.1 we present the whole sample analyzed (i.e. units of analysis; VRs; proposed exercises).

Table I.1: The sample analyzed in the dimensions of textual units, VRs and proposed exercises

GENERAL SUBJECT	SUBJECT GRADE_No	Thematic units	VRs in content presentation sections	Proposed exercises
MATHEMATICS	Math_Gr9 (GYMNASIO)	2	8	7
	Math_Gr11_Common Core (GENERAL LYKEIO)	22	43	59
	Math_Gr11_Scientific (GENERAL LYKEIO)	3	5	
	Math Gr12_Positive & Tech direction (GENERAL LYKEIO)	4	7	19

TOTAL MATHEMATICS		29	63	85
PHYSICS	Phys_Gr9 (GYMNASIO)	8	11	2
	Phys_Gr11_Common Core (GENERAL LYKEIO)	15	36	32
	Phys_Gr11 Positive & Tech direction (GENERAL LYKEIO)	5	20	-
	Phys_Gr12 Positive & Tech direction (GENERAL LYKEIO)	15	54	24
TOTAL PHYSICS		43	121	58
TOTAL: 72 Thematic units, 184 VRs, 143 exercises				

Data analysis

We explored each thematic unit looking for parts in which the text could be divided and could function as appropriate units of analysis. We realized eventually that reading and consequently understanding a thematic unit is related to the reasoning process and the argumentation developed in this part of the chapter. Argumentation is considered to be developed through everyday examples, empirical evidence, explanations, proofs, definitions, calculations, type of visual representations and the co-deployment of VRs and MsoR.

As a result our analysis followed the next steps:

In the first step, we identified parts of the text in the thematic unit that could be considered as logical elements that develop a form of reasoning. These parts may correspond to one or more sentences and the accompanying visual representations that were characterized as *modes of reasoning* (MsoR). Stacey and Vincent (2009) used this term for analyzing explanations in mathematics textbooks. Our use of MsoR is that there are parts that state a syllogism crucial for the development of argumentation in the whole thematic unit. Therefore, MsoR in this study has a rather functional than structural character and characterizes a phase in the development of thematic unit's argumentation and an act in promoting conceptual understanding. Three criteria guided our analysis in terms of MsoR: (a) the function of each mode of reasoning in the argumentation developed in the thematic unit as a whole; (b) the data or evidence on which the reasoning is based; and (c) the nature of the warrants or the backings that support the reasoning explicitly or implicitly. One of our main concerns was that the categories be appropriate for coding both science and mathematics texts, since evidence and warrants may have a different ontology in each disciplinary field. The technique of systemic networks (Bliss, Monk & Ogborn, 1983) has been adopted not only as a form of representing our scheme of categories, but also as an analytic tool. This means that it has been used throughout the analysis for the organization and the continuous re-arrangement of categories and sub-categories as they emerged during the analysis of mathematics and science texts.

Basic elements of the thematic units are the visual images (e.g., graphs, tables, photographs), so, in the second step our focus of analysis was on the genre of VRs. In the third step we analyzed the co-deployment of visual representation and the mode of reasoning or how the visual representation supports the mode of reasoning. We consider this co-deployment as part of the argumentation developed in the thematic unit.

All the codes emerged from the analysis were negotiated among the two researchers. Within a feedback loop, the codes were revised, eventually reduced to main categories, and checked in respect to their reliability until a satisfactory degree of certainty/consensus was obtained. As a result, the final scheme of categories and subcategories of MsoR, VRs genre and the categories of co-deployment of VRs and MoR were formed. The final systemic networks produced were considered to be a generalized scheme appropriate for both mathematics and physics texts on periodicity. This analytic process offered an insight into the argumentation development in each thematic unit and in particular, how it is formed as a synthesis of different MsoR and different ways of co-deployment of MsoR & VRs. Finally, this approach gave us the opportunity to explore possible links between the argumentation and the conceptualization process, which may influence the reader's understanding.

In the fourth step of our analysis, the emerging schemes supported the quantification of our data (MsoR), VRs' genre and VRs & MoR met in all the thematic units we had selected for analysis. Quantitative findings show the frequencies that the categories met across thematic units and across mathematics and physics.

In the fifth step of our analysis we analyzed the 143 proposed exercises by using an adaptation of the analytical framework of MoR described above.

FINDINGS

From our data analysis seems that argumentation in the content presentation sections in a school text is the combined outcome of three components: the modes of reasoning (MsoR), the visual representations (VRs) and the co-deployment of VRs and MsoR. In the first part we present each category and subcategory separately of each component separately. In the second part we present results of our quantitative analysis of all the above categories and subcategories. In the third part we illustrate typical forms of argumentation in mathematical and physics textbooks related to the concept of periodicity based on two examples. In the last part we present our analysis of the proposed exercises by adjusting the initial analysis to the analytical framework presented above.

Modes of Reasoning (MsoR)

The final scheme of MsoR categories appears in Figure I.1 as a systemic network (Bliss et al., 1983). Four mutually exclusive categories of MsoR were identified: empirical, logical-empirical, nomological, and mathematical.

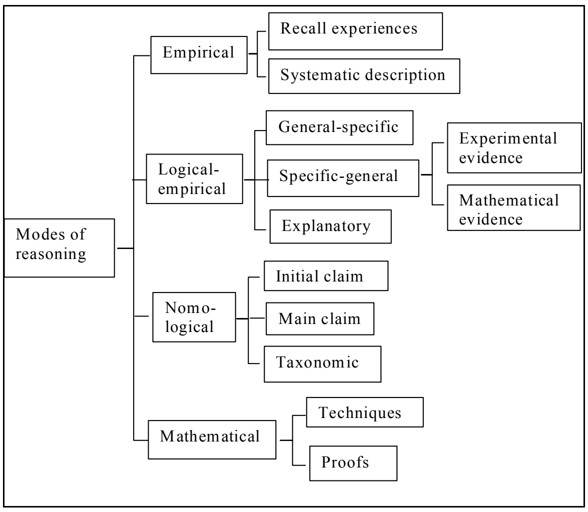


Fig. I.1. The systemic network (Bliss et al., 1983) of MsoR.

Empirical MsoR

The empirical category includes parts of texts where the reasoning is based on evidence that either recalls one's everyday life experiences or is based on a systematic description or demonstration of an experimental activity or everyday life situation.

Logical-empirical MsoR

The logical-empirical category includes parts of texts in which the connections of evidence included in the unit to logical elements are more prevalent, and examples and specific situations that are second-hand experiences are used to reach or exemplify general statements. This category is further discerned into three subcategories: general-specific, specific-general, and explanatory. The general-specific MoR refers to applications of a law or general statement (e.g., definition) stated previously in the unit. The specific-general MoR includes reasoning that presupposes analysis and interpretation of instances of scientific situations (phenomena) or organized empirical data in order to make the proper conclusion or a generalization. In this MoR, two sets of reasoning were identified, depending on the kind of evidence or data being developed: experimental evidence, when the reasoning is based on instances of experimental activities that are mostly displayed using visual representations, and mathematical evidence, when the reasoning is based on mathematical representations (e.g., graphs, organized arithmetical data) or on mathematical models (e.g., the unit circle). The explanatory MoR includes text that tries to explain theoretical ideas or exploit invented situations in order to explain a particular phenomenon. This type of reasoning shares common characteristics with the specific-general MoR but is differentiated due to its function in the argumentation developed in the thematic unit, which means that its role is mainly to explain why a theoretical issue is true rather than to infer from the particular to the general case.

The aim of this MoR is to provide everyday examples of a certain periodic motion type to motivate readers' interest. In mathematics, this MoR mostly involves exemplification with numerical data of a mathematical formula of a definition that has been proved or given in the previous MoR. In such examples, readers are normally practicing a technique but are also expected to appreciate the range and scope of the generality of the theorem proved before. This MoR's general function is to help readers gain a personal sense of a definition or general principle in science (Anderson & Smith, 1987) and mathematics (Bills et al., 2006).

An example of specific-general MoR based on experimental evidence is from a physics textbook's thematic unit on the simple pendulum. The text is accompanied by a visual representation (i.e., images of the pendulum in different positions simultaneously). The text reasons as follows: *When the particle is at the equilibrium position, the string is vertical. If the object is not at the equilibrium position, it oscillates between the end positions B and C. The forces that determine its movement are the weight and the force that is exerted by the string.* The caption explains why the pendulum performs this type of periodic motion: *In every position, the weight* W_2 *component moves the particle to the equilibrium position.* Every statement in this unit of analysis supports a process of generalization. This MoR is an integral part of the nature of science itself because scientists usually rely on it to lay the ground for new research and to support or refute their research hypothesis (Norris & Philips, 2003).

Nomological MsoR

The nomological category includes parts of texts that are general statements (e.g., theorems, laws, or definitions). Kublikowski (2009) said that this category is characterized by its logical and rhetorical emphasis on the pragmatic and persuasive role of definitions considered as an integral part of argumentation. We discerned three subcategories in this MsoR: initial claim, main claim, and taxonomic. Initial claim uses statements based on a known law or a general principle that function mostly as the starting point of reasoning in the argumentation developed in a thematic unit. In most cases, reasons are not offered to justify these statements; however, they play a dominant role in the argumentation developed. Main claim uses statements formulated as the result of previous logical acts reasoning central to a thematic unit. Taxonomic includes statements that clarify categories of periodic behaviors.

An example of the initial claim MoR is taken from the 2nd year upper secondary school mathematics unit "Graphing the sine function". In order to define the period (T) and study the sine function, the text states: *Since the function f(x) = sinx is periodic with period 2π, it is sufficient to study it in an interval that has length 2π, e.g. [0, 2π]*. The text continues by using this claim to graph the sinx function.

A main claim MoR encapsulates the main points of the unit; it is usually recorded in the text in a distinctive way (e.g., bold font) in order to attract the readers' attention. This reasoning can be considered to be the outcome of logical acts made before in other MsoR; for example, it may follow a descriptive empirical or specific-general MoR. The main claim MoR is a definition or general principle that establishes specific, conceptual aspects of periodicity (e.g., definitions of linear harmonic oscillations or periodic functions). Our analysis indicates that this MoR plays a significant role in the development of argumentation in mathematics and science textbooks.

A characteristic example of the taxonomic MoR is taken from a physics textbook. The text presents the uniform circular motion as a part of a wide category of motions that are called periodical. This MoR asks readers to categorize, describe, and classify different types of periodic phenomena in a hierarchical manner.

Mathematical MsoR

The mathematical category includes parts of text based on applying mathematical relationships and techniques in both mathematics and science; the emphasis here is on the instrumental and functional character of reasoning. We discerned two subcategories: techniques and proofs. In the techniques MoR, reasoning is based on techniques, usually well-known mathematical ones; for example, sketching a graph from a given table of values, solving an equation graphically, or providing a sequence of numerical operations. In the proofs MoR, reasoning has a pure deductive character; in the selected thematic units, it is based mainly on algebraic manipulations; for example, formal mathematical demonstrations (Stylianides, 2009).

An example of the proofs MoR is provided from a physics thematic unit (i.e., energy in the simple harmonic oscillation). The text proves that the total energy is constant even though the amounts of the two forms of mechanical energy, kinetic and potential, change periodically over time: *The total energy of the system in a random position is given by the relation* $E = K + U$. *Hence,* $E = ½\, m\omega^2 A^2 cos^2\omega t + ½\, m\omega^2 A^2\, sin^2\omega t = ½\, m\omega^2 A^2\, (cos^2\omega t + sin^2\omega t) = ½\, m\omega^2 A^2$. In proofs, the aspects of periodicity are usually met in the form of symbolic representations of the sinusoidal functions.

Visual representations' (VRs') genre

In terms of the genre of the VRs, five main categories are defined: the *photographs*, the *natural drawings*, the *schematic representations, graphs and Tables*. Although these categories are content free, the sub-categories are related to the features of the content concerning periodicity.

So, *photographs* have been found to present either instances of either every day life examples or natural periodic phenomena or technological devices performing periodic motions.

The category of *naturalistic drawings* has been found to present either naturalistic drawings of every day periodic phenomena or other no periodic images.

The category of *schematic representations* has been found to present periodic motions circular or elliptical variations; mechanical oscillations; or synthetic variations or non periodic.

The category of graphs has been found to present sinusoidal curves or other periodic curves or non periodic. The last category is the tables.

Co-deployment of VRs & MoR

Four mutually exclusive categories concerning the role of VR in reasoning have been identified: (a) the *embodying category*: when the VR and/or its caption supports the reasoning by presenting in a bodily form an aspect of the MoR is related to; (b) the category of VR as the *starting point:* in this case the reasoning starts from the specific visual representation and is further developed; (c) the category of VR as the *fundamental* tool of reasoning. In this case, the reader needs to be based on the VR throughout the reasoning process; (d) the category of *product* VR: in this case the VR functions as the final product of the reasoning developed.

Qualitative analysis of the argumentation developed in two thematic units

In this section, we attempt to reveal the development of argumentation as a synthesis of a sequence of MsoR in two thematic units: mathematics and physics. We consider that VRs' genre and the co-deployment of VRs & MsoR are playing a fundamental role in the argumentation as well. A key aim is to reveal how argumentation unfolds in the articulation of the new knowledge.

The mathematical text is from the subject of trigonometry, and its thematic content is "Graphing the sinx function." The physics text is from the subject of oscillations, and its thematic content is "Defining the linear harmonic oscillation". These thematic units study periodic variations in different contexts. In mathematics, it is the counterclockwise rotation of a point M(x,y) on the unit circle; in physics, it is the motion of a body that is attached to an ideal spring. We chose these thematic units for the following reasons. In both texts, the periodic variations are modeled by the sinusoidal curve. In mathematics, this curve models the variation of the y-coordinate of the point M(x,y); in physics, the same curve models the time variation of the displacement of a body that oscillates vertically with the help of an ideal spring. Both texts are addressed to the same student group (i.e., 2^{nd} year upper secondary school). Moreover, both thematic units exhibit a typical form of knowledge presentation for periodicity in the two subjects. According to the Greek curriculum, the mathematics unit precedes the physics unit.

Argumentation in the mathematics text (Figure I.2) starts by defining the period T of the periodic motion and by asking readers to recall that sinx represents the y-coordinate of the point M(x,y) on the unit circle.

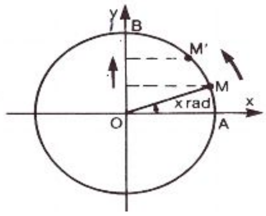
Graphing the function sinx

Since the function f(x) = sinx is periodic with period 2π, it is sufficient to be studied in an interval that has length 2π, e.g. $[0, 2\pi]$. As we already know, sinx represents the y-coordinate of the point M(x,y) on the unit circle. Therefore, it is sufficient to examine how y-coordinate of point M varies, when it rotates around the trigonometric circle towards the positive direction, starting from A.

Nomological, initial claim MsoR

We notice that:

- As x varies from 0 to $\pi/2$, the point M moves from A to B. Therefore, the y-coordinate increases, thus the function sinx is strictly increasing in the interval $[0, \pi/2]$.



Similarly, we find that the function sinx:

— strictly decreases in the interval $[\pi/2, \pi]$,

strictly decreases in $[\pi, 3\pi/2]$,

and strictly increases in $[3\pi/2, 2\pi]$.

- The function has
- a maximum value on $x = \pi/2$, sinx = 1 and
- a minimum value on $x = 3\pi/2$, sinx = -1.

Logical-empirical, specific-general, mathematical evidence MoR

The results are summarized in the following table:

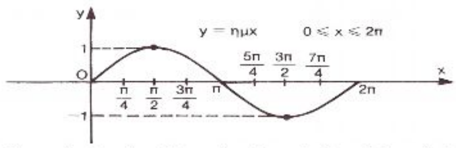
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
ημx	0 ↗	1 ↘ μεγ.	0 ↘	−1 ↗ ελαχ.	0

In order to sketch the graph of this function, we need a table value. According to what we already know, we complete the next table.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
ημx	0	$\sqrt{2}/2 \approx 0{,}71$	1	0,71	0	−0,71	−1	−0,71	0

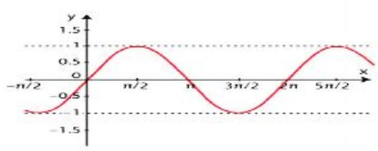
We represent the couples (x, sinx) with points on the Cartesian plane and we join them with a continuous line.
In this way, the following graph of the sinx function on the interval $[0, 2\pi]$ is produced.

Mathematical, techniques MoR



Since the function f(x) = sinx is periodic, with period 2π, the curve has the same shape in the intervals $[2\pi, 4\pi]$, $[4\pi, 6\pi]$ etc. and in the intervals $[-2\pi, 0]$, $[-4\pi, -2\pi]$ etc.
This is depicted in the following graph of the **sinusoidal curve**.

Nomological, Main claim MoR



Finally, we know that opposite angles have opposite sines. Therefore, for every $x \in \mathbb{R}$, we have $\sin(-x) = -\sin x$. This means that the function f(x)=sinx is odd and hence its graph has a point symmetry on O(0,0).

Logical-empirical, explanatory MoR

Fig. I.2. Analysis of mathematics text (Andreadakis et al., 2012. pp. 75-77).

We consider these as *nomological* MsoR because these parts of the text are given statements that function as *initial claims* on which the next MsoR are based. The text continues by asking readers to follow the y-coordinate of the point M(x,y) as it rotates around the unit circle. Readers have to interpret that the increase or decrease of the y-coordinate corresponds to an increase or a decrease of the function y = sinx.

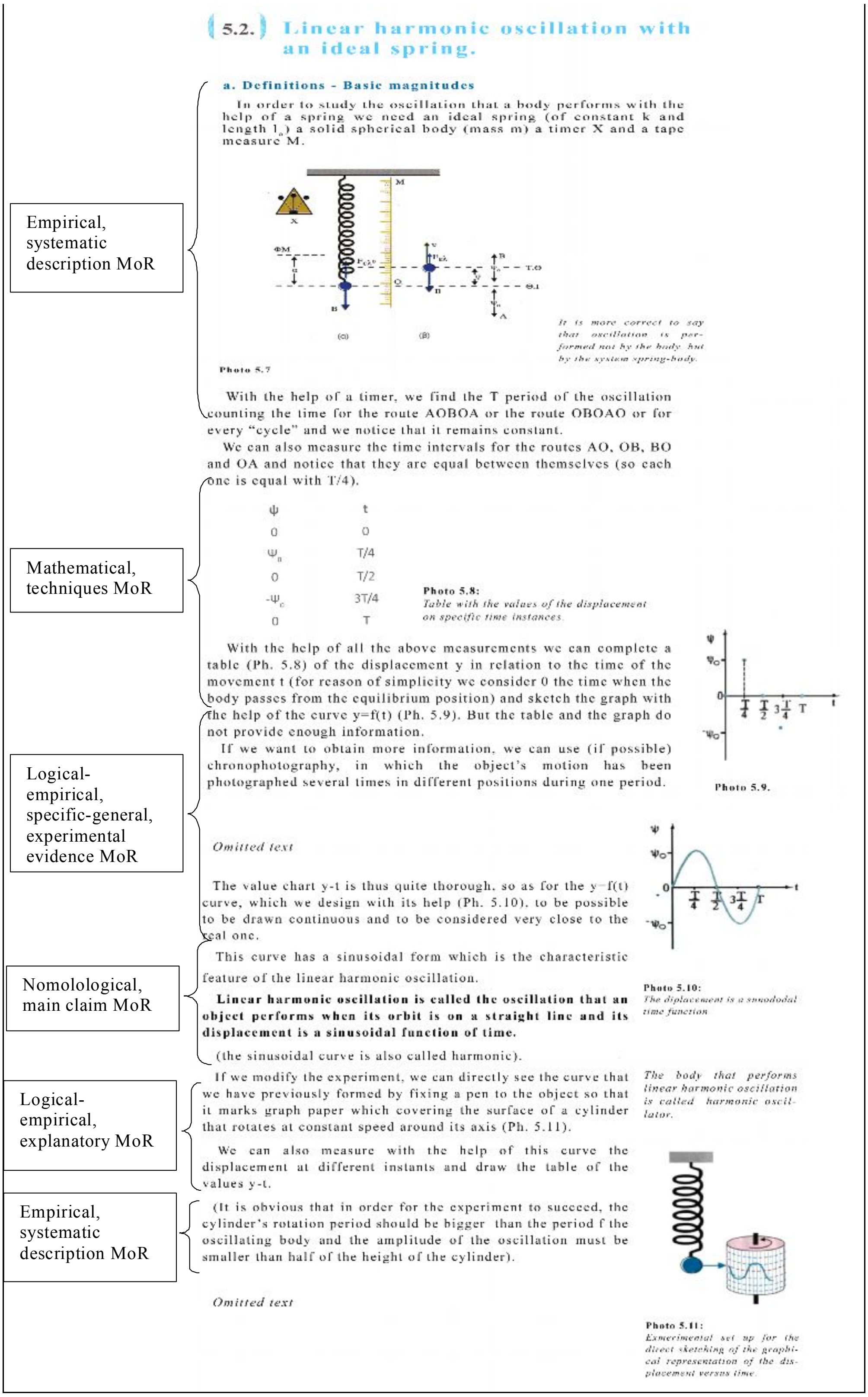
We consider this MoR as *logical-empirical* and particularly *specific-general*, which is based on *mathematical evidence* identified on the unit circle representation. The argumentation then employs *mathematical techniques* (i.e., graph the sinusoidal function), and the text then defines the curve as sinusoidal. This is characterized as *nomological* and functions as a *main claim* MoR. The text concludes by visually representing a sinusoidal curve and then argues the odd property of the sinusoidal function. This is considered as a *logical-empirical, explanatory* MoR because it uses the basis of semi-empirical evidence (i.e., the visual representation) to explain the function's property.

In this thematic unit there are five VRs (one schematic representation of circular variation, the trigonometric circle, two tables and two sinusoidal graphs). In the first paragraph the reasoning is based on the trigonometric circle. The role of this mathematical model in the particular mode of reasoning is *fundamental* since the reader must reflect on this VR in order to comprehend the reasoning developed in the main text. The two tables are the *starting points* of the mathematical MoR while the first sinusoidal curve is the *product* of this MoR. The second sinusoidal curve plays a fundamental role in the explanatory MoR that concludes the argumentation of this thematic unit.

Argumentation in the physics text (Figure I.3) starts with a *systematic description* MoR that is produced by part of a text and an accompanying visual representation where an enacted experience of an experimental activity is presented. The justification proposed is made with the help of empirical measurements of the movement of a spring with the main goal of defining its period.

The argumentation continues by producing a table and the corresponding graphical representation categorized as a *mathematical technique* MoR. Next, even though it is based on experimental methods, the sketching of the sinusoidal curve has an obvious intention of generalizing the outcomes of this experimental activity since the reasoning points to the fact that the sinusoidal graph represents a linear harmonic function. Therefore, this form of reasoning is characterized as *logical-empirical, specific-general* MoR based on *experimental evidence*. This MoR sets the foundation for formalizing the physical event in later studies**.** The definition of linear harmonic oscillation emerges as the result of the previous inferences and is characterized as *nomological* MoR functioning as a *main claim*. This is a *logical-empirical, explanatory* MoR that ends the argumentation by addressing the necessary conditions for the success of the experiment. The last MoR is characterized as *empirical* and provides a *systematic description* of the situation under consideration.

In this thematic unit there are five VRs (one *schematic representation* of mechanical oscillations; one table, two graphs (one not finished and one sinusoidal) and a *schematic representation of synthetic periodic* variations. In the first paragraph the reasoning starts from the VR and ends with the table. The two graphs are the product of the experimental mode of reasoning while the sinusoidal graph plays a fundamental role in the nomological main Claim MoR. The last graph plays a embodying role in the empirical mode of reasoning that concludes the argumentation of this thematic unit.


5.2. Linear harmonic oscillation with an ideal spring.

a. Definitions - Basic magnitudes

In order to study the oscillation that a body performs with the help of a spring we need an ideal spring (of constant k and length l_o) a solid spherical body (mass m) a timer X and a tape measure M.

It is more correct to say that oscillation is performed not by the body but by the system spring-body.

Photo 5.7

With the help of a timer, we find the T period of the oscillation counting the time for the route AOBOA or the route OBOAO or for every "cycle" and we notice that it remains constant.

We can also measure the time intervals for the routes AO, OB, BO and OA and notice that they are equal between themselves (so each one is equal with T/4).

ψ	t
0	0
ψ_o	T/4
0	T/2
$-\psi_o$	3T/4
0	T

Photo 5.8: Table with the values of the displacement on specific time instances.

With the help of all the above measurements we can complete a table (Ph. 5.8) of the displacement y in relation to the time of the movement t (for reason of simplicity we consider 0 the time when the body passes from the equilibrium position) and sketch the graph with the help of the curve y=f(t) (Ph. 5.9). But the table and the graph do not provide enough information.

If we want to obtain more information, we can use (if possible) chronophotography, in which the object's motion has been photographed several times in different positions during one period.

Photo 5.9.

Omitted text

The value chart y-t is thus quite thorough, so as for the y=f(t) curve, which we design with its help (Ph. 5.10), to be possible to be drawn continuous and to be considered very close to the real one.

This curve has a sinusoidal form which is the characteristic feature of the linear harmonic oscillation.

Linear harmonic oscillation is called the oscillation that an object performs when its orbit is on a straight line and its displacement is a sinusoidal function of time.

(the sinusoidal curve is also called harmonic).

Photo 5.10: The diplacement is a sinusodal time function

If we modify the experiment, we can directly see the curve that we have previously formed by fixing a pen to the object so that it marks graph paper which covering the surface of a cylinder that rotates at constant speed around its axis (Ph. 5.11).

The body that performs linear harmonic oscillation is called harmonic oscillator.

We can also measure with the help of this curve the displacement at different instants and draw the table of the values y-t.

(It is obvious that in order for the experiment to succeed, the cylinder's rotation period should be bigger than the period f the oscillating body and the amplitude of the oscillation must be smaller than half of the height of the cylinder).

Omitted text

Photo 5.11: Exmerimental set up for the direct sketching of the graphical representation of the displacement versus time.

Fig.I.3. Analysis of physics text (Alexaki et al., 2012, pp. 204-206.

Issues emerged from the qualitative analysis of the argumentation developed in two thematic Units

Our analytic approach on these two thematic units reveals how argumentation is produced across two directions: the sequence of MsoR and the unfolding of the conceptual aspects of periodicity. We noticed initially that, for the study of both periodic motions, the specification of the period is central; however, it is accomplished differently. In physics, the period needs to be measured empirically; in mathematics, it is taken as granted on the basis of the claim that emerged from the definition of sinx as a periodic function.

The sinusoidal curve is considered the main object of study in both thematic units. The reasoning route, however, follows different paths based on the different nature of tools and actions employed. In mathematics, the curve is the product of mathematical evidence-based reasoning on the variation of the y-coordinate of the point M(x,y) as moving around the trigonometric unit circle, a theoretical functioning entity of mathematics; in physics, the curve is the product of experimental evidence-based reasoning, which stems from the study of the motion of a body on an ideal spring, and the means through which the linear harmonic oscillation is defined. Hence, the goal in each thematic unit seems to be different: in mathematics, the process of drawing the sinusoidal graph is the main task; in physics, this graph functions as the necessary condition in order to define linear harmonic oscillation. The definition of the linear harmonic oscillation (main claim MoR) is validated by an additional experiment through which readers will be persuaded and convinced about the scientific claim.

Analysis of the sequence of MsoR in these two thematic units raised two key issues. The first issue relates to ontological differences concerning the concept of *evidence* in mathematics and physics. By tracing the different paths of reasoning in these two settings, our analysis offers insights into the different rules that prevail in each disciplinary community concerning evidence-based reasoning. Experimental evidence in the physics textbook is strengthened by the collection and use of new data; mathematical evidence in the mathematics textbook is considered as beyond controversy and, as a result, no further strengthening is needed. The second issue relates to pragmatic considerations on the text understanding in relation to the scientific argumentation discourse. In the physics text, the claim that the curve produced by the experimental activity is a sinusoidal curve appears arbitrary. A surface view is that readers must simply take the mathematical product (e.g., the image of the sinusoidal curve) and use it as a tool in the process of meaning making of the physics text. Our analysis indicates that if readers do not pass through certain MsoR resulting in the sinusoidal curve then important conceptual or logical elements may be missing. It appears that, in the context of physics, the way mathematical tools are used in reasoning is elliptic and the understanding of the text is based on hidden speech acts (e.g., explaining that the produced curve is the sinusoidal curve). On the other hand, the mathematics text follows a linear and coherent reasoning in terms of the logic, using consistently the mathematics discourse. Furthermore, in the mathematics context, efforts to bring tools and materials concerning reality — empirical MsoR — seem to be neglected.

The role of VRs is central in each thematic unit since they play an important role in the argumentation developed. Images of periodicity share common characteristics. For example, the sinusoidal curve and the circular periodic variations appear in both texts. Physics text provides additionally images of oscillatory motions. In both texts the role of all images is fundamental or the staring point or the product of a MoR. Only in physics text an image plays an embodying role in the MoR developed. This role shares empirical characteristics and brings the reader closer to the periodical behaviour. The issue that this VR represents a synthetic periodic motion (circular &

oscillatory) adds to its contribution to readers' understanding of the notion of periodicity.

Results of our quantitative analysis on the dimensions of MsoR, VRs' genre & co-deployment of VRs & MsoR

Modes of Reasoning

To provide an overview of the nature of the MsoR in the different subjects in the Greek textbooks cornering periodicity, we present some simple quantitative measures. Table I.2 shows frequencies for certain type of MsoR in all thematic units. Particularly, we counted all MsoR identified in all thematic units for each subject. The proportion of each type of MoR reported was calculated for the total number of the modes of reasoning identified in each subject (Mathematics N= 138; Physics N=231). For example, we identified N=14 Mathematical proofs in mathematics texts and 13 in physics texts. Hence the proportion of this MoR in mathematics is 14/138*100=10.14% and in physics is 13/231*100=5.63%.

Table I.2: Results of the quantitative analysis in the dimension of MsoR

Modes of reasoning			Subjects	
			Mathematics N=138 (%)	Physics N=231 (%)
Empirical	Recalling experiences (E1)		0.00	1.73
	Systematic description (E2)		2.90	13.85
	Sum of E1 & E2		*2.90*	*15.58*
Logical-empirical	General-specific (LE1)		6.52	12.55
	Specific-general	Experimental evidence (LE2exp)	0.00	8.23
		Mathematical evidence (LE2m)	23.19	9.96
	Explanatory (LE3)		5.07	6.49
	Sum of LE1, LE2exp, LE2m & LE3		*34.78*	*37.23*
Nomological	Initial claim (N1)		15.22	9.09
	Main claim (N2)		31.16	26.84
	Taxonomic (N3)		0.00	3.03
	Sum of N1, N2 & N3		*46.38*	*38.96*
Mathematical	Techniques (Mteck)		5.80	2.60
	Proofs (Mpr)		10.14	5.63
	Sum of Mteck & Mpr		*15.94*	*8.23*

The *empirical* category of MsoR has very low appearance in mathematics (2.90%) and low appearance in physics (15.58%). Particularly, the *systematic description* MoR seems to be usual in physics only in lower grades.

The *logical-empirical* category of MsoR seems to have almost the same appearance in math and physics. Despite this general outcome we notice some differences in subcategories of this MoR among subjects. For example, the *general-specific* subcategory seems to be more common in physics (12.55%) than in mathematics (6.52%). This MoR general function is to help the reader gain a personal sense of a definition or general principle. Its significance is acknowledged in science (Anderson & Smith, 1987) and mathematics (Bills et al., 2006) communities. The *experimental evidence* appears only in physics (8.23%). This mode of reasoning is an integral part of the nature of science itself, since usually scientists rely on it to lay the ground for new research and to support or refute their research hypothesis (Norris & Philips, 2003). The *mathematical evidence* is met mostly in mathematics (23.19%) than in physics (9.96%) texts.

The *nomological* MsoR have a high appearance in both subjects, with a little lead in mathematics. It seems that two nomological categories are high privileged in mathematics and physics texts. These are the *main claim* and the *initial claim* that considered as common reasoning techniques in both subjects. The *taxonomic* MoR is met only in physics texts. Referring to neighbouring concepts of periodicity in a taxonomic way could help the reader develop a flexible and functional knowledge, but this seems not valued as a significant rational action in both subjects.

Finally, the mathematical category of MoR (*mathematical techniques* and *proofs*) has with very low proportional appearance in math and physics (15.94% and 8.23% respectively).

We place each category of MoR in a spectrum of logical acts from sensory perceptions/experiences (empirical & logical –empirical MsoR) to abstract logical thinking (nomological & mathematical MsoR) (Fig. I. 4). Hence, we notice that physics keeps a balance between sensory perceptions and abstract thinking (52.81% and 47.19%) respectively. Mathematics favours the abstract logical acts (62.32%) than the sensory ones (37.68%).

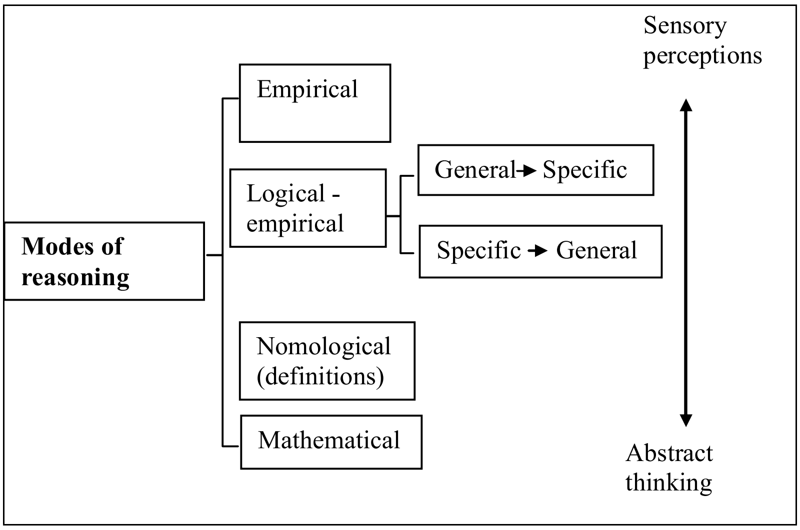


Fig. I.4: The spectrum of logical acts (MsoR) from sensory perceptions to abstract thinking.

Some of the differences identified above are due to the nature of each epistemological field (e.g. the absence of experimental evidence in mathematics, or the higher appearance of mathematical MsoR in mathematics than in physics; the balance between sensory perceptions and abstract thinking in physics). But some other differences may indicate policies in each community (the privilege of abstract MsoR in the argumentation developed in mathematics or in the main role of nomological MsoR in both subjects).

The Visual Representations' (VRs') genre

In mathematics we met 63 VRs in the 29 thematic units analyzed while in physics we met 121 VRs in the 43 thematic units analyzed.

The average number of VRs in each thematic unit is 2.17 mathematics (63/29=2.17) and 2.8 in Physics (121/43). The relation of each VR with a mode of reasoning is 0.45 (63/138) in mathematics and 0.52 in physics (121/231). In general, the differences among the subjects are very small.

Table I.3 presents the percentages of the categories of VReps in terms of their genre.

Table I.3: Results of the quantitative analysis in the dimension of VRs' genre			
		Mathematics N=63 (%)	Physics N=121 (%)
Photos (images)	Every day or natural periodic phenomena (ph1)	0.00	13.22
	Scientific devices performing periodic motions	0.00	4.96
Naturalistic drawings	Every day periodic motions	3.17	13.22
	No periodic motions	0	3.30
Schematic representations	Circular periodic variations	46.03	5.79
	Mechanical oscillations	0	19.83
	Synthetic periodic variations	0	4.13
	Non periodic	7.94	4.13
Graphs	Sinusoidal curves	17.46	18.19
	Other periodic graphs	6.35	2.48
	Non periodic	0	8.26
Tables		19.04	2.48

It can be noticed that *photograph*s of any type of periodic images are completely absent in Mathematics textbooks while almost three out ten (31.4%) of all VRs in physics are either *photos or naturalistic images* of periodic motions. It is common policy in Mathematics textbooks to avoid using photographs (that represent particular instances) and prefer to use images that convey a generality (Herbel-Eisenmann & Wagner, 2007).

On the other side, the *graphs of sinusoidal functions* are used in both Mathematics and Physics texts and they are considered as the main graphical models of the periodic behavior. The case of graphs that represent a repeated but non-periodical behaviour appears only in physics (in the case of damped oscillations). It could be very helpful for students to compare images that represent a periodical behaviour with images that represent a repeated but non-periodical behaviour. In general, this type of images usually called non-examples of a notion (Bills et al., 2006) and influence the discernment of concepts. In our case, non-examples of periodical motion could used to clarify boundaries between neighboring features of periodicity.

In the case of schematic representations the main image of periodic variation is the circular one represented by the trigonometric circle. One out of two VRs in mathematics falls in this category. In Physics the most common periodic models are mechanical oscillations.

Finally, charts and tables are used more often in the Mathematics textbooks (almost 20%) than in Physics textbooks (almost 2.5%). Usually this type of representations organizing the information given in the main text or provide specific values of a function or a phenomenon.

The co-deployment of VRs & MoR

Our analysis indicated 65 cases in mathematics and 128 in physics. Some times a VR is related with one or more MsoR. In Table I.4 presents the frequencies of the categories of the function of VRs in reasoning in a tabular format as they are met in the Mathematics and Physics texts.

Table I.4: Results of the quantitative analysis on the dimension of co-deployment VRs & MsoR

Categories	Mathematics N=65 (%)	Physics N=128 (%)
Embodying (represents in a bodily form)	7.70	30.49
Starting point of reasoning	33.84	19.53
Fundamental tool in reasoning	37.00	30.47
Product of reasoning	21.54	19.53

We can notice the prevalence of the *embodying function* of VRs in reasoning in Physics texts comparing to the Mathematical ones. In this case the VR and/or its caption support the mode of reasoning by presenting a concrete situation. This brings the reader more closely to periodic phenomena.

Images play a *fundamental* role in mathematics and physics with a little lead in the latter subject. Images as the *starting point* of reasoning are mostly in math than in physics. Finally, VRs are almost equally the product of reasoning in mathematics and in physics.

Issues emerged from our quantitative analysis of MsoR, VRs' genre & co-deployment of MsoR & VRs

Our quantitative analysis provides a quick overview of the most and less preferable reasoning practices adopted in mathematics and physics. This reveals differences in the nature of each epistemological field and policies in each community.

The authors in Mathematics texts seem to avoid reasoning practices that are based on corporeal MsoR, or VRs that refer to sensual perceptions (photos and/or naturalistic drawings) while rarely the VR functions as a concrete situation (i.e. plays an embodying role) on the MoR developed in text. The notion of periodicity is treated mostly by nomo-logical & mathematical MsoR (62%); almost catholically (96%) by schematic representations or graphs or tables while VRs rarely (8%) play an embodying role in the reasoning developed. All the above indicate that the emphasis in math is on abstract ways of thinking and reasoning.

On the other side, the authors in physics texts support sensual aspects of learning since three out of ten images are photos naturalistic drawings of periodic motions and the same proportion play an embodying role in the MoR developed. Besides, they keep a balance between reasoning that is based in sensory experiences and in abstract logical thinking.

The fundamental images of periodic motions are circular in mathematics (the trigonometric circle) and oscillatory in physics. There is a consensus in both subjects on the periodical curve image since in both subjects two out of ten graphs are sinusoidal curves in math and physics.

Finally, almost all thematic units 'build' their argumentation around a main claim, nomological MoR. This adds to our perception that all thematic units in both subjects are argumentative and not descriptive texts. This indicates that argumentation is considered as a central activity in both subjects. We do not know yet if the teachers in the two communities realize the central role of argumentation in their teaching practice.

Textbooks proposed exercises

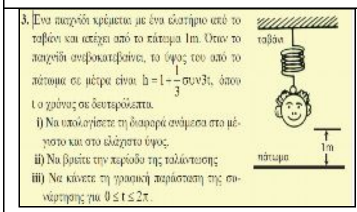
Textbooks influence teaching not only with the way they present new knowledge but with the type of proposed exercises as well (Mesa, 2004).

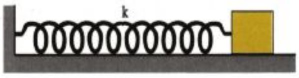
The proposed exercises are analyzed according to their *demands on exercises, the type of VRs* and their *contextual aspects.*

We analyzed 85 exercises in mathematics and 58 in physics. In the initial analysis the categories of demands on exercises were: *identify a property on a mathematical or experimental model*, *make simple* or *synthetic calculations*, *produce or proving a mathematical or scientific outcome*. By taking into consideration the reasoning practices demanded by the students in each subcategory on exercises demands we see that the case of *identifying a property on a mathematical or experimental model* as corresponding *to a Logical- empirical, specific-general MoR.* The case of *making calculations* (simple or synthetic) corresponding to a pure mathematical MoR. The case of *produce or proving a mathematical or scientific outcome* corresponding to combination of Logical-empirical (if a VR is present), Nomo-logical & mathematical MsoR.

In terms on the VRs presented on the exercise two categories are defined: *absence* and *existence*. The second category was further analyzed according to the following subcategories: *School type* VR (we include schemes and figures), *real life* and *graphical representations*. In terms of the *contextual* aspects that was supported with the exercises the following categories emerged: *context free*, *pseudo-context* (where the exercises on the surface seemed to be about real world problems and situations, but actually they had little connection to the real world) and *real life* context.

In Table I.5 we exemplify our analysis on two typical examples taken from the subject of mathematics and physics.

<table>
<tr><td colspan="2">Table I.5: Examples of the qualitative analysis of the proposed exercises</td></tr>
<tr><td>
</td><td>3. A toy hangs with a spring from the ceiling and is 1m from the floor. When the toy goes up and down, its height from the floor counted in metres is h=1+1/3sin3t, t is the time in seconds.
(i) Calculate the difference between the maximum and the minimum height (of the toy).
(ii) Evaluate the period of the oscillation.
(iii) Sketch the graph of the function for $0 \leq t \leq 2\pi$.</td></tr>
<tr><td>Demands
(i) Identify properties
(ii) Make simple calculations
(iii) Produce a mathematical model
VRs
Existence
Context
Existence/pseudo-context</td><td>Reasoning practices demanded by the students
(i) Logical –empirical, specific-general MoR
(ii) Mathematical MoR
(iii) Combine nomo-logical & mathematical (techniques) MsoR.</td></tr>
</table>

 Σώμα μάζας 0,2Kg ηρεμεί πάνω σε λείο οριζόντιο επίπεδο δεμένο στο ελεύθερο άκρο ελατηρίου σταθεράς 20N/m. Αν το σώμα απομακρυνθεί λίγο από τη θέση του κατά τη διεύθυνση του άξονα του ελατηρίου και αφεθεί στη συνέχεια ελεύθερο: α) να δείξετε ότι θα εκτελέσει Γ.Α.Τ., β) να βρείτε την περιοδό του.	A 0,2Kg body is in balance in a smooth horizontal surface tied from the free side of a spring having 20N/m constant. If the body is removed towards the axis of the spring and is left free: a. show that a Linear Harmonic Oscillation is going to be performed b. find its period.
Demands (a) & (b) produce a scientific model *VRs* Existence *Context* Existence/pseudo-context	*Reasoning practices demanded by the students* Combine Nomo-logical, Mathematical & Log.-empirical MsoR

The counting of frequencies of appearance of the final produced schemes in the form of systemic network for the dimension of the demands on the proposed exercises are presenting in the following table. In the last column we present the demands according to the demanding or reasoning skills.

Table I.6: Results on the quantitave analysis on the demands of exercises

Demands of exercises	MATHEMATICS No = 85 (%)	PHYSICS No = 58 (%)	According to the new framework
Identify a property	21*	19*	Logical- empirical, specific-general MoR (math or experimental)
Calculate	21*	37*	Mathematical MoR
Prove a mathematical or a scientific outcome	79*	71*	Combining Mathematical & Log. –empirical & nomological MsoR

* The sum is more than 100% since some times there are more than one tasks on the same exercise.

VRs' existence (either in exercice presentation or as an outcome) is very low in both subjects. Besides, 90% of the proposed exercices in mathematics are context free. This rate is lower in physics. Even physics rarely base exercises on real- life context.

Table I.7: Results on the quantitative analysis on the existence of VRs and real-life context

		MATHEMATICS	PHYSICS
Existence of VRs		MATHEMATICS N =85 N= 14 (17%)	PHYSICS N=58 N=11 (19%)
Contextual aspects		MATHEMATICS N =85 (%)	PHYSICS N=58 (%)
Context free (abstract)		90	59
Existence	Pseudo-context	9	26
	Real life	1	16

Issues emerged from the analysis of the proposed exercises

In general there do not exist many differences in the reasoning skills demanded by the students in the two subjects. The most common reasoning practice in both subjects is the mathematical MsoR (appears in the majority of exercises in math and physics). Furthermore, most cases of textbooks proposed exercises tasks are demanding a combination of Mathematical & Log.–empirical & nomological MsoR.

VRs are rare in mathematics and physics. Almost all mathematics exercises are context-free while only 16% of the proposed exercises in physics are using the real life context.

CONCLUDING REMARKS on PART I

The present study explored reasoning and argumentation in Greek mathematics and physics texts in specific topics related to the notion of periodicity. Reasoning has been investigated through the logical act created by a part of the text of a thematic unit, a choice that differentiates our use of modes of reasoning from that of Stacey and Vincent (2009). We adopt the position that textbooks aim to introduce their readers to the conceptual aspects of scientific and mathematical knowledge and persuade them for their value.

Particularly, we consider that the argumentation developed by an author in a school textbook is a combination of analytical and rhetorical arguments, employed to persuade the reader (who in our case could be a student or a teacher). Furthermore, the function of the visual representations in relation to the reasoning developed in the physics or mathematics text is investigated. In this study we argue that the VRs' genre and the co-deployment of VR and MsoR influence the argumentation developed in a school text and consequently had a potential impact on how the notion of periodicity is realized. Although the inherent logic of a concept presentation has been acknowledged as important for a teaching plan (Koponen & Nousiainen, 2012), in our study we have elaborated how the interrelationships of argumentation and conceptualization are developed.

Inductive content analysis was applied on 72 thematic units and 184 Visual Representations (VRs). Coding schemes of categories and subcategories of MsoR, VRs' genre, and the co-deployment of VRs and MsoR were produced. The main categories of MsoR are empirical, logical-empirical, nomological, and mathematical MsoR. We argue that each MoR plays a different role in conceptualizing aspects of periodicity. The empirical MsoR attempt to direct readers' attention to recall experiences on periodic motions in real-life phenomena. The logical-empirical MsoR link empirical evidence to general outcomes; therefore, studying the dynamic features of certain instances of periodic phenomena could lead to understanding their general characteristics or vice versa. The nomological MsoR indicate the epistemological and ontological aspects of periodicity aimed to be learnt in each subject. Finally, the mathematical MsoR usually provide support to scientific claims (e.g., proving that the mechanical energy of an oscillation is always constant) or mathematical claims (e.g., proving that sinx + cosx is equal to $\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$), where representations of periodicity are mostly symbolic in nature (e.g., sinx in mathematics texts or Asinωt in physics texts). The main categories of VRs' genre are photos, naturalistic drawings, schematic representations, graphs and tables. The role of VRs in MsoR could be embodied (when VR supports the MoR by presenting a concrete situation), the starting point or the product of a MoR and fundamental (when the reader needs to be based on the VR throughout the reasoning process). The photos and naturalistic drawings of periodic motions help readers to visualize everyday periodic phenomena, while the schematic representations and the graphs support more abstract ways of thinking.

Quantitative analysis of the sequence of MsoR in two thematic units (one from physics and one from mathematics) raised three key issues. The first issue is related to the ontological differences concerning the concept of *evidence* in mathematics and physics. Experimental evidence in the physics textbook is strengthened by the collection and use of new data, while mathematical evidence in the mathematics textbook is considered as beyond controversy and, as a result, no further strengthening is needed. The second issue is related to *pragmatical* considerations involved, important for the text understanding, in relation to the scientific argumentation discourse. For example, in the physics text, the claim that the curve produced by the experimental activity is a sinusoidal curve appears arbitrary. Our analysis indicates that if readers do not pass through certain MsoR resulting in the sinusoidal curve then important conceptual or logical elements may be missing. The third issue is the *central role of VRs in the argumentation* developed in each thematic unit in both subjects.

The results of our quantitative analysis raise important issues as well. Almost all thematic units in mathematics and physics 'build' their argumentation around a main claim, which is a nomological MoR. This supports our perception that *all thematic units in both subjects are mostly argumentative and not descriptive texts*. The authors in Mathematics texts seem to avoid logical acts that refer to sensual perceptions (empirical MsoR, photos, and/or naturalistic drawings), while rarely the VR concretizes (represents in a bodily form) the MoR developed in text. On the other side, the authors in physics texts support sensual aspects of learning since three out of ten images are photos and naturalistic drawings, that means that they represent in a bodily form the MoR they are related to. Besides, they keep a balance between reasoning that is based in sensory experiences and in abstract logical thinking. The above outcomes indicate different practices in mathematics and physics textbooks. Physics try to *bring the notion closer to readers' sensual experiences,* while in mathematics *there is an emphasis on abstract ways of thinking and reasoning*. Although some of the above outcomes are due to the different epistemological nature of mathematics and physics (e.g., the fact that physics brings every day conceptions about periodicity closer to readers' experiences) some other indicate the different practices adopted by the two communities. For example, it is common practice in Mathematics textbooks to avoid using photographs (that represent particular instances) and prefer to use images that convey a generality (Herbel-Eisenmann & Wagner, 2007). This practice is purposeful, since authors believe that these images do not support a 'proper mathematical' argument. Although recent research has stressed the decisive and prominent role of bodily actions and gestures in students' elaboration of elementary concepts, as well as abstract mathematical knowledge (Núñez 2000), this is neglected in Greek math textbooks. Finally, there is a consensus in both subjects on the use of *periodical curve image,* since two out of ten VRs are sinusoidal curves in both subjects.

Furthermore, our analysis indicates that the absence of any of the categories of MsoR and VRs could result in missing conceptual elements of periodicity that are important for understanding. In this way we argue for the importance of argumentation in the conceptualization process.

The proposed exercises mostly require of students to develop synthetic logical acts (Logical-empirical, nomological and mathematical MsoR). The qualitative comparison on the reasoning practices adopted in the two textbook sections 'content presentation' and 'proposed exercises' raises some important issues as well. The issues concern the *existence of VRs* and *the use of real life examples* as the context of the exercises. Particularly, although the role of VRs is fundamental in the 'content presentation sections' in both subjects their presence in the 'proposed exercises sections' is relatively low. In the 'content presentation sections' physics use real life examples in a form of modes of reasoning or images, this is not the case of the 'proposed exercises sections'. The real life context is mostly absent in physics exercises, while it is completely absent in mathematics. So, students when asked to solve the proposed exercises rarely are engaged in interpreting VRs. As a result, *modelling activities with real-life periodic motions* seem to be neglected in both educational communities, while mostly abstract conceptions of periodicity are stimulated by the solutions to exercises and problems in the given sample.

In this case, the balance between reasoning that is based in sensory perceptions/experiences and in abstract thinking identified in the 'content presentation sections' in physics does not been preserved in the 'proposed exercises sections'.

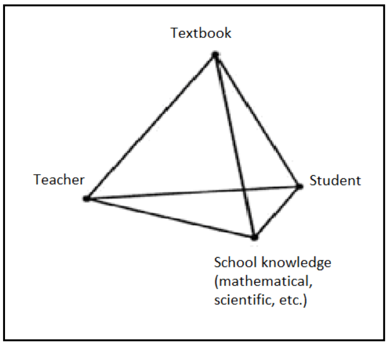
Ch. 3

PART II

"Focusing on Undergraduate Students' & Secondary Teachers' Reasoning Practices about the Notion of Periodicity"

INTRODUCTION

Rezat (2006) argues that mathematics textbooks should not be a subject to analysis detached from its use. The tetrahedron model illustrates the complex relationships among the four constituents of the didactical situation—teacher, students, school content knowledge (mathematical or scientific) and artefacts (in our case textbooks) that mediate the pedagogical practice in school The whole tetrahedron is a comprehensive model of the didactical situation in the sense that it encompasses the interplay of artefacts (textbooks), teachers and students in the construction of school knowledge. All four constituents are interlaced beyond the possibility of separation.



In the first part (Part I) of this report our focus was on the argumentation developed in textbooks. In this part (Part II) our motivated research question is "*how students' knowledge and teachers' practices are influenced by the argumentation and reasoning practices adopted in textbooks*".

In this part of the report we focus on undergraduate students' argumentation practices when they had to interpret secondary level texts and VRs on periodicity and teachers' pedagogical practices when teaching specific thematic units on periodicity and when they are handling students' justifications.

THEORETICAL BACKGROUND

Teachers' & students' dependence on textbooks

Students in order to understand a school text they have to realize the argumentation developed by the author. Love and Pimm (1996) note, that although the implied relation between the reader and the text is inherently passive, "*the most active invitation to any reader seems to be to work through the text to see why the particular 'this' is so*" (p. 371). Chi and her colleagues' (Chi, deLeeuw, Chiu & LaVancher, 1994) research in the science context highlighted the importance of the argumentation developed in textbooks in the meaning-making process. Specifically, they argue that students, in order to understand the text material, generate self-explanations, since even quality expositions require the reader to fill in substantial details.

As students construct their understandings of the nature of mathematics or science they will draw to different extents on the textbook, on their teachers' speech and actions and on their previous experiences. On the other side, where teachers are insecure in their own subject knowledge they are likely to rely heavily on the forms of definition and argumentation that are provided for them in published resources. Haggarty and Pepin (2002) note that in England, while students themselves make relatively little use of textbooks, their teachers use them extensively in planning lessons. In this way textbooks have a strong influence, whether direct or indirect, on students' understanding.

Argumentation and students' conceptualization

Despite the numerous studies on argumentation and reasoning, there is limited understanding of how argumentation practices could influence students' conceptualization.

In the case of mathematics, Lin and Yang's study (2007) is an example of an attempt to make such links by analyzing the interrelationship between students' reading comprehension of mathematical proofs and their content knowledge. Douek's study (1999) is another attempt that aimed at detecting productive links between context-related argumentation and conceptualization.

Besides, a number of recent research projects examined the impact of argumentation on conceptual understanding in science. For example, Jime´nez-Aleixandre and Pereiro-Mun˜oz (2002) who found that the involvement of students in argumentation and decision making about environmental management resulted in them becoming knowledge producers, not because they created new knowledge, but because they applied knowledge to practical contexts, combined ecological concepts, and integrated conceptual knowledge with values. Venville & Dawson (2010) explored the impact of classroom-based argumentation on high school students' argumentation skills, informal reasoning, and conceptual understanding of genetics. The importance of the findings are that after only a short intervention of three lessons, improvements in the structure and complexity of students' arguments, the degree of rational informal reasoning, and students' conceptual understanding of science can occur. The findings also provide evidence to show that training students in argumentation and having them participate in whole-class argumentation about socio-scientific issues resulted in them being able to produce more rational written arguments.

Teachers' pedagogical practices and reasoning

The importance of understanding and improving students' reasoning has been greatly stressed as teachers' instrument to enhance both teaching effectiveness and learning abilities. Recent approaches have focused on the structure, on the completeness and the nature of arguments, their logical properties and their content.

According to reform documents, teachers are expected to teach proofs and proving in school mathematics and engage in inquiry and argumentation activities in science.

In the mathematics context, Lampert (1990, p. 32) describes pedagogical teachers' practice in traditional school classrooms as follows:

> Teachers tell students whether their answers are right or wrong, but few teachers engage students in a public analysis of the assumptions that they make to get their answers. Even when teachers give an explanation rather than simply stating a rule to be followed, they do not invite students to examine the mathematical assumptions behind the explanation, and it is unlikely that they do so themselves […].

On the other side, in a pedagogical practice that is based on learning mathematics as participating in the social setting where argumentation and reasoning is a central activity in the classroom…

> [participants] arguing about what is mathematically true; they move around in their thinking from observations to generalizations and back to observations to refute their own ideas and those of their classmates … they put themselves in the position of

> authors of ideas and arguments; in their talk about mathematics, reasoning and mathematical arguments (ibid).

Nicol & Crespo (2006) study investigates how four prospective teachers interpret and use textbooks while learning to teach mathematics during university coursework and practicum teaching. Results indicate that prospective teachers had varied approaches to using textbooks ranging from adherence, elaboration, and creation. Factors influencing how they engaged with texts include their practicum classroom setting, access to resources, and their understanding of mathematics. Pre-service teachers' attempts to modify textbook lessons raised pedagogical, curricular, and mathematical questions for them that were not easily answered by reference to the textbooks or teacher's guides.

Tabach, et al., (2010) examined the position of secondary school teachers with regard to verbal proofs. Fifty high school teachers were asked to evaluate given justifications to statements from elementary number theory. Teachers are not aware of students' preference for verbal justifications while about half of the teachers rejected correct verbal justifications. They claimed that these justifications lacked generality and are mere examples.

In the scientific context, Lawson' (2004) study focused his attention in the case of hypothetico- deductive reasoning in order to test alternative explanations. He argues that science teachers' effective instruction mirrors the practice of science where students confront puzzling observations and then personally participate in the explanation generation and testing process – a process in which some of their ideas are contradicted by the evidence and by the arguments of others.

Koponen & Nousiainen (2012) argues on the logical order of argument, where the ontology, facts and methods all have their proper places and are related to each other correctly so that inferences can be made in a reliable and justified manner.

Mahidi (2013) examined the role of topic in the hierarchical organizations between knowledge of teachers and textbooks. The hierarchical organizations of teachers are more comparable to textbooks for the topic of Biot-Savart law than the Ampère's law. On the other side, it was observed that the knowledge arrangements of Ampère's law were more hierarchical, while the knowledge organizations of Biot-Savart were more clustered.

Finally, Oherman & Lawson (2008) argue that science and mathematics teachers, curriculum developers and textbook authors owe it to students to more carefully explicate the similarities, differences, and limitations of knowledge-generation processes in both fields, particularly the meanings of the terms proof, disproof, hypotheses, predictions, theories, laws, conjectures, axioms, theorems, and postulates, so that students have a better chance of avoiding misconceptions and/or confusion about how these aspects of science and mathematics work. The reason for researchers' argument is that many science and mathematics teachers have too little understanding of the knowledge generation process in their own discipline, much less adequate understanding of the process in their sister discipline to help students acquire meaningful understanding.

Our epistemological view on teaching is that it consists of generating and keeping in movement contextual activities which are situated in space and time and heading towards a fixed pattern of reflective activity incrusted in the different school cultures (in our case mathematics, science and engineering). This movement has three essential characteristics: the object (that, in our case, is the notion of periodicity) it is not a monolithic or homogenous object. It is an object made up of layers of generality and these layers will be more or less general depending on the characteristics of the cultural meanings of the fixed pattern of activity in question. An example of this is the kinaesthetic movement of a child that plays in a playground swing by forming a periodic motion in a certain time interval and the graphical representation of the above movement as height-time variation. The layers of generality are noticed in a progressive way by the student. The learning process consists of finding out how to take note of, or how to perceive these layers of generality (Radford, Cerulli, Demers, & Guzmán, 2004).

Purpose of PART II of our study, Research Questions and Research Activities

We divide this part of our study in ***PART IIa*** and ***PART IIb***.

In ***PART IIa*** we focus on undergraduate students' meaning making of textual and visual elements of school texts on periodicity. We analyze their justifications on certain tasks related to aspects of the notion and we try to detect potential links between conceptualization and argumentation. The title of this part of our research is ***"Undergraduate students' justifications in the process of making sense of textual and visual elements on periodicity"*** The undergraduate students are in scientific direction fields who passed a national exam in order to attend their undergraduate studies. Part of their exams where thematic units on periodicity (e.g.. mechanical and electrical oscillations). Besides, during their undergraduate studies all students in scientific direction fields encounter aspects of periodicity (usually in in their first year Calculus and Fourier analysis courses). Fourier analysis is a prerequisite course for studying signal processing in the fields of Informatics and Electronics. Thus, for all the participants, periodicity is considered as an important scientific notion not only for their academic studies, but for their professional life as well.

In this part of our study we take the position that understanding the notion of periodicity and its properties involves creating a coherent framework where ideas and educational practices in different school subjects are meaningful at an individual level. By adopting the perspective that periodicity, as an abstract notion, is realized through specific situations where it takes its meaning (Radford, 2013) we design three different research activities (case studies) where different aspects of the notion are involved. Our main interest is how students perceive periodic motions and its graphical representations. Our resources are mostly mathematics, physics and engineering secondary school textbooks.

Our main research questions in this part of our study are:

- How do undergraduate students meaning making of verbal and visual elements is related to specific justifications (reasoning practices)?
- If and how we can detect potential links between conceptualization and the above justifications?

In order to accomplish our aims we designed the following three research activities where students in open-ended questionnaires are responding in the following tasks:

Activity 1: *Interpreting and connecting the textual and the visual components of school texts on periodicity.*

Activity 2: *Making sense out of periodical and non - periodical motion graphs.*

Activity 3: *Relating prospective teachers' explanations and their levels of content awareness.*

We consider that the above three research activities could provide us evidence on students' justifications (reasoning practices) when interpreting verbal and visual texts on periodicity in three contexts (physics, mathematics and technology). Activity 1 is developed in a physics context (the texts and the VRs are taken from physics Greek textbooks). Activity 2 provides a mathematical context by pointing only on the mathematical models of periodical behaviours. Activity 3 provides a technological context where the synthesis of the above knowledge is necessary in order to provide sufficient explanations of a text on car mechanics. Students' answers could help us to detect productive links between reasoning practices and conceptualization.

In ***PART IIb*** our focus is on secondary mathematics and science teachers' pedagogical practices when teaching specific thematic units on periodicity. The title of this part of our study is: ***"Teachers' pedagogical tools when teaching periodicity"*** Particularly, we are seeking to discover how educators in the various disciplines institutionalize their students' knowledge on aspects of periodicity and how they use texts' inherit logic when teaching aspects of periodicity. In order to investigate this general issue we designed and conducted two research activities. Both were designed in a unified way for science, mathematics and engineering educators. In this manner, we expect to identify differences in educators' practices when they teach aspects of the notion of periodicity.

Teaching aspects of periodicity in mathematics and science involves images of instances (or aspects or properties or models) of the notion. These representations in a school text are expressed either visually (e.g. pictures, diagrams or maps) or symbolically (e.g. equations or formulae). The role of images of a common notion in different teaching practices remains under investigated. We consider that the representations of the notion of periodicity are cultural resources which act as bearers of distributed intelligence (Pea, 1993) and that they carry, in a compressed way, socio-historical experiences of cognitive activity and artistic and scientific standards of inquiry (Lektorsky, 1995). These ubiquitous mediating structures both organize and constrain educators' teaching practice and provide to students a specific, conceptually structured space to think (Radford, 2013). Are the educators in the different disciplines adequately equipped to handle the above issues? This is another aspect we try to investigate in this study (Activity 1-questionnaires).

Our research activities and the relevant research questions are as follows:

Activity 1- questionnaires "Teachers' justifications and fundamental images on periodicity"

We designed a questionnaire with open-end questions and placed emphasis on the role of visual representations (VRs) in educators' teaching practices when aspects of the notion are presented and how they handle specific students' justifications (PART IIa).

Our main research questions are:

- Which images are fundamental in their teaching practices?
- How do they argue against students' misunderstanding of the periodic behaviour of two graphical representations?

Activity 2- interviews: "Teachers' judgments when modifying textbook argumentation"

We interview educators on the role of textbook argumentation in their teaching practice and ask them specifically about the role of the everyday examples they use when introducing aspects of periodicity:

- What is the role of everyday examples in their teaching practices?
- Do they follow the knowledge organization in texts in specific thematic units related to the notion of periodicity?

A common task in Act1& Act2 "Teachers' suggestions"

In both activities, we ask teachers' suggestions on how they could contribute to their students' development of a unified way of understanding periodicity.

PART IIa

Documentation of undergraduate students' thinking and reasoning

METHODOLOGY

The participants

The participants in all research activities were 288 undergraduate students from Greek University and Technological Institutions and took part in three research activities. In Table II.1 we present the number of participants in the different departments in each research activity (we implement three research activities). The participants filled open-ended questionnaires.

Table II.1: The number of participants from the different departments in different activities

Institutions	Departments	No of Activity	No of participants
Technological Institutions	Electronics	1	41
		2	43
	Informatics	2	31
	Structural Engineers	1	13
	Mechanical Engineers	1	16
		3	86
Universities	Bioinformatics	2	39
	Mathematics	2	19
Total 288 participants			

In this report we present the methodology and the results of the three research activities that took place in this study.

Activity 1: "Interpreting and connecting the textual and the visual components of school texts on periodicity"(context: physics)

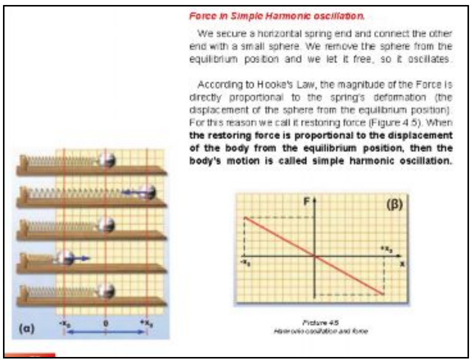


Photo 1

A research questionnaire has been developed with open-ended questions on the topic *"Study of the motion of the linear spring"*. This questionnaire was distributed to 70 undergraduate engineering students from four different departments in two different Technological Institutions (13 structural engineering, 16 mechanical engineering and 41 from the field of Electronics). The students were in the second semester of their studies. This topic was very familiar to them since in the first semester was pat of their studies. The text (Photo 1) and the VR (Photo 2) that they had to interpret was from their secondary school year textbooks.

Task 1

The book extract which is presented in Picture 1 shows an experimental arrangement that refers to the study of the movement of the spring and is included in the 3rd Grade Secondary School Science book. The fist task (extract from Physics, Grade 9 textbook), was about Hooke's Law (i.e. the magnitude of the restoring force is directly proportional to the spring's deformation) as a sufficient condition of spring's Harmonic motion.

The argumentation of this thematic unit was as follows: The text starts with an *empirical, systematic description* MoR where Picture (4.3.a) playing a fundamental role. Then an *initial claim, nomological MoR* was referring to Hook's Law. The VR (linear graph) played a fundamental role in this MoR as well. The text concludes its argumentation with a Logical-empirical, specific-general mode of reasoning since it requires for the reader to connect the two VRs that interpret visually the *Nomological, main claim mode of reasoning* (definition of SHO).

Our research question was: "*How do students understand the argumentation developed in the text?" "How do students explain conventions on the VRs (e.g. the negative values of F)?"*The questions addressed to the students were *"(a) what does the author want to show to his readers? How would you explain the existence of negative values of F as shown in Picture 4.5(b)?"*The students in order to respond in the questions they had to understand the argumentation developed in the text and interpret appropriately connections between the MsoR and VRs and their conventions as well.

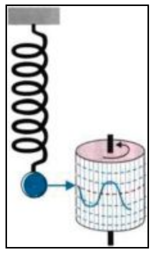


Photo 2

Task 2

In the second task students are required to connect different graphical representations of the same periodical phenomenon (springs' motion). The question was as follows: *"The two graphs in Photos 1 and 2 refer to the same phenomenon. What relationships are depicted in each one of the two graphs? What are their differences? Analyze and justify your answer".*

The linear relation F-x and the sinusoidal curve are depicted different aspects of the spring's periodic motion.

FINDINGS

Task 1: 67 out of the 70 participants responded in Task 1. From the 67 participants only 35% (N = 24) enriched the textual information or used other arguments than the one mentioned in the text. The rest mostly copied the MoR presented in the text or made false interpretations of the modes of reasoning find in the text (e.g. imagine that there is someone who applies a force on the spring all the time). Text enrichment or make a clear and correct explanation of the argumentation developed in the text is considered as an important parameter of content knowledge awareness (see results on Activity 3). These students interpreted appropriately the mathematical signs according to the physical Law (Hook's Law) and clearly acknowledged the important role of VRs in this thematic unit.

A typical response is: "*The author's aim is to introduce the students to simple harmonic oscillations by showing them* [he does not use the term 'prove', so he realizing the informal role of the Logical-empirical argument!] *that the restoring Force applied on the spring is proportional to springs' displacement as Hook's Law defines. The author tries to convince the readers with the use of the representations, these help the students to understand better ... Due to the restoring force the spring moves from the position* $-x_o$ *to* $+x_o$ *and continues this motion periodically" (st49, elec).*

St. 49 made a clear explanation of the role of the Logical-empirical mode of reasoning in the text argumentation, and realized the fundamental role of the two VRs in this MoR. Moreover, she visualizes the periodical motion of the spring from -xo to +xo all the time which has a result on the periodical variation of F(x) as Figure II. 1 shows.

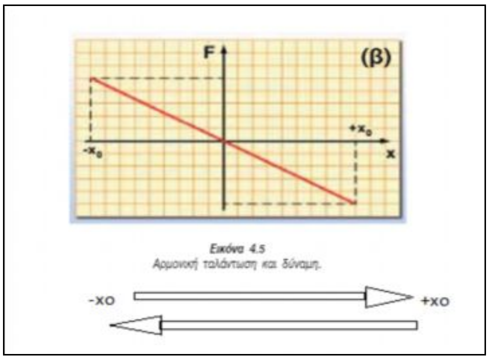


Fig. II.1: St 49's dynamic image of the periodical variation

Only 6 students or 9% of the participants who responded in this task made a convincing argument (a sufficient explanation) for the existence of negative values for the Force F on the graph. In this case, students managed to connect their (mathematical) knowledge on vectors with their scientific knowledge on Hook's Law.

One explanation was as follows "*F = - Kx, hence the force was proportional to the displacement x but their signs are opposite, when the force is positive the displacement is negative and the other way around*" (st_30 elec). We classify this reasoning as nomological since the student applies the formal definition of Hook's Law. Another type of sufficient explanation was as follows: *"the displacement takes negative values when it is compressed and positive values when it is elongated. The force acts against the above spring's displacement since it always tends to bring the spring into its physical position (equilibrium position). When the spring is compressed (negative x) acts in the positive direction in order to bring the spring in the equilibrium position and the opposite* (st11_str)". We classify this reasoning as Logical-empirical, specific-general, while the fundamental role of the VR is acknowledged by the participant. In this case he manages to connect functionally the scientific situation with his empirical observations.

Some students made a partial explanation by arguing that this is the reason that we call the force "restoring", works against the springs' deformation or refer to the "opposite vectors direction". Finally, some of irrelevant responses were: *"as many times the spring oscillates the more negative values the force takes st48 elec" "Force is a vector so instead of the value we take into consideration and the direction of the force (st 19 mech.)", the negative values are because the Force is decreasing (st 20 mech.)", or "the negative values are due to the frictions"(st._65 Elect).*

Task 2: This task was requiring students to make connections among representations of the same periodic phenomenon. The analysis of students' responses in Act 2/Task 2 (identifying that the Hook's law display the linear relation of the x-F while the sinusoidal curve display the relation x-t and both representations are referring to different aspects of the same phenomenon) provided the following information.

Only 37 of the 70 participants replied in this task.

From those who participated, the majority interpreted the different graphs due to the different spatial arrangement of the spring-in horizontal and vertical position. They made naive generalizations based on some empirical observations. Only some of them managed to connect the graphical representations as representing different aspects (mathematical relations) of the same phenomenon (N=13 of the 70 participants). The last outcome addresses students' compartmentalization in knowledge. Below we present an example of a student who realized the different graphical representations as representing different aspects (mathematical relations) of the same phenomenon:

> "*Photo 1 [represents the relation] (F, displacement) & Photo 2 [represents the relation] (displacement, time) or in Photo 1 we have F = -Dx while in Fig. 2 we have F=-Fmaxsin(ωt+φ)*" (st 17_mech).

St17 identifies that the linear relation of the x-F and the sinusoidal curve (relation x-t) both are referring to different aspects of the same phenomenon.

All the students who made a sufficient explanation in the second question of Task 1 responded successfully in Task 2. This is an indication that students there are potential links between deep content knowledge and sufficient justifications.

Table II.2: Percentages of students' responses in the category of the 'connection different VRs

Categories	Connect the different graphical representations of the same phenomenon (N=37 responded in the question)
Not make any connections	45.9% (N=17)
Interpret one of the two successfully	18.9% (N=7)
Connecting both as graphical representations of the same phenomenon that are depicted different mathematical relations	35.1% (N=13)

Activity 2: Justifying periodical and non-periodical graphical behaviours (context: mathematics)

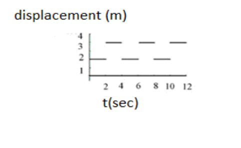
METHODOLOGY

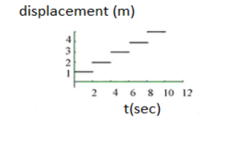
This study aims to explore how undergraduate students in mathematics and engineering professions make sense out of graphs representing periodic and repeated but non-periodic motions. In this study, making sense out of graphs means interpreting graphical features and describing a situation that could be represented by them. The data was collected by means of a questionnaire with open-ended questions administered to 132 participants (undergraduate students).

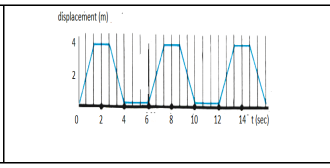
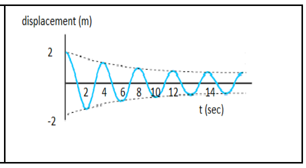
The participants from two different University Departments (Mathematics and Bioinformatics) and two different Technological Institutions Departments (Electronics and Informatics). 19 students were studying Mathematics, 70 were studying Informatics (39 Bioinformatics and 31 Informatics) and 43 were studying Electronics.

Table II.3: The graphs in the initial report*

Graph 1 (Buendia & Cordero, 2005)	Graph 2 (Buendia & Cordero, 2005)	Graph 3 (Greek mathematics textbook)	Graph 4 (Greek physics textbook)





*In the current report we restrict our analysis on graphs 1 & 4.

The questionnaire was completed in one teaching hour during a mathematics course in the case of the engineering students and during a course in mathematics education in the case of students in mathematics. Four graphs were given to the students that all represent displacement in meters versus time in seconds. Table II.3 shows the four graphs and the resources used.

Graph 1 represents non-continuous periodic motion while Graph 4 represents non-periodic motion. Two tasks ere given to the students referring to each graph separately. Task 1: *Does this graph represent a periodic motion?* (Identify periodical behaviour). *Justify your answer* (justification). In this task, students are asked to focus on how the repetition is accomplished in order to distinguish the periodic from the non-periodic motions, as well as justifying their response. Task 2: *Provide an example that could be described by this particular graph* (exemplification). In this task the students were asked to assign to each graph a motion that could be represented by it.

In the current report we restrict our analysis on graphs 1 & 4.

FINDINGS

Students' participation in the three tasks

Since students participated voluntarily in this activity we consider that the same students' high or low participation in specific tasks over other tasks signifies students' readiness to be involved with them.

We identified low participation of students in the task on justifying which graph represents periodic or non-periodic motion and high participation in the task of providing an example that could be described by this particular graph. This fact is surprising since the 'exemplification task' is considered to be more time-consuming and effort-required than the 'justification task' one. Even though not all examples provided by the students were appropriately describe the graph we cannot ignore the fact that students prefer to develop logical- empirical justifications by connecting a mathematical (logical) model to their personal experiences of periodical behaviours than developing formal justifications (that mostly require the use of nomological syllogisms).

Table II. 4: Students participation in tasks

Tasks	Graph 1	Graph 4
	N=132	N=132
Identifying periodical behaviour.	113	114
Justification of response.	58	64
Exemplification	84	102

Hence, difference in students' participation in tasks could provide us with information concerning students' preference to work in tasks requiring visualization and imagination than reasoning and abstracting, and using their own experiences that justifying at an abstract level.

Identifying periodical behaviour (Does this graph represent a periodic motion?)

Two categories emerged from the analysis of this task: the graph represents a *non-periodic motion;* and the graph represents a *periodic motion*. Almost three out of four students identified periodicity in Graph 1. Graph 4, which represents a repeated but a non-periodic motion, seemed to confuse students a lot since almost seven out of ten considered it to be a periodic graph. We can explain this students' misunderstanding since in physics texts the above graph describes a decreasing oscillation which under some restrictions is considered to be a periodic motion.

Table II. 5: Identifying periodical behaviour

Categories	Graph 1	Graph 4
	113 Participants	114 Participants
Non-Periodic	23.89	32.46
Periodic	76.11	67.55

Students' Reponses on the justification task

When we analyzed students' justifications the following categories emerged: naive justifications; Logical-empirical, general-specific & specific-general, and nomological. We have to notice that we analyze true or false justifications.

Naïve justifications that are based on naive beliefs about the periodic notion. An example of naive justifications for the case of Graph 1*"It is not periodic because there is not any harmony in the graph"* (st59_elec). It is interesting that the periodic graph (Graph 1) cause students more naive justifications than the non-periodic one.

Some characteristic examples of Logical-empirical, general (the graph)-specific (a concrete example) students' justifications are: (Graph 1) "*the body of the graph diverges from the starting point of motion and then always returns within 4 seconds, therefore the graph is periodic (st99_elec*); and (Graph 4) "*it is periodic because it represents the motion of the swing*" (st129_inf). This indicates students' need to set up a background for their justifications.

Logical –empirical, general -specific is the case that the students generalize by pointing on specific observations on variations on the graphical representations "Graph 4: *It is periodic but we can see that as the time passes it dwindles and we are led to a standstill"* (st101_elec); or "*It is a periodic motion that decreases (its amplitude diminishes) all the time*" (st68_elec); ". Although st101 expresses a doubt he/she still names the motion 'periodic'. In st68 response the decreasing amplitude 'fits' with periodicity. Some students used continuity issues as a warrant to take the stance that Graphs 1 is non-periodic. For example, st19_math notices: "*I do not know if this graph preserves a periodic behaviour because in its second position it has different values from left and rig*ht".

Finally, we consider as *mixed justifications* when we could identify elements of nomo-logical (constant period, f(x+T) = f(x), decreased harmonic oscillation, etc.) and Logical-empirical MsoR (identify elements on graphical representations) in participants' responses. An example is: *"(Graph 4) it is periodic since it is a decreased harmonic oscillation with a constant period"* (st76_elec.). Most participants used parts of definitions of periodic motions taken from their physics texts while only one (!) used the formal definition of periodic functions taken from their math texts. Particular, st6_math reasons on what is periodic by using the 'definition of periodic functions' in the case of Graph 1. To our surprise, in the case of Graph 4, the same student changes her argument as follows: "*It is periodic since any* *sinusoidal function is periodic*" since she thinks that the particular graph represents "a sinusoidal function". Many students used this argument in order to justify that Graph 4 represents a periodic graph. We consider this argument as a mixed justification.

Table II. 6 present percentages of students' type of justifications. We notice the privilege role of the Logical-empirical, from general to specific in both Graphs but mostly in the case of Graph 4 (four out of ten participants used an example in order to justify their response. A common example was the playground swing which was introduced as an example of periodic motion in their physics and mathematics textbooks. The case of the periodic graph causes a lot of naïve justifications in relation to the non-periodic graph.

Table II. 6: Students' justifications

Categories	Graph 1 N=58 (%)	Graph 4 N=64 (%)
Naïve justifications	17.24	6.25
Log.-Emp, from general to specific	25.86	39.06
Log-Emp, from specific to general	22.41	31.25
Mixed (Nomological & Log. empirical, specific-general)	34.48	23.44

Finally, in the case of Graph 1 many students tried to synthesize definitions of periodic motions (nomological) and observations on the graphical representations and this helped them to provide a correct answer. The same type of justification does not help them to realize gaps in their reasoning. The influence of the reasoning developed in their physics class was dominated their understanding.

Students' responses on the exemplification task

Graph 1

Creating a motion example of a piece-wise continuous function is very difficult but a few students managed to provide examples that could satisfy all the graphical features in this graph. Particularly, only 7 participants provided examples that are satisfying the graphical conventions on Graph 1. A typical example of *enriched repeated motions* in the case of Graph 1: "*ascending and descending jumps between uneven steps* (st1_math)". In this case, all participants used their kinesthetic experiences of 'jumping' or 'climbing stairs' in order to respond successfully in this task.

Graph 4

Many students used the swing example for responding in the case of Graph 4. This example was a typical example in their physics classes.

Physical tools' motions provided the context used by most students to translate the graphs to situations. Bodily actions were used by 25% of participants in Graph 1 and only 8% of participants in Graph 4. We interpret this result that most students consider that human actions are very difficult to model Graph 4 motion graph so they have changed the context of their example from bodily actions to physical tools' motions or vibrations of natural tools. More than one out of four students used examples of *vibrating natural objects* (e.g. sea waves) in describing the case of Graph 4. The graphical image resemblance with travelling sinusoidal waves was the reason to use them as the context of their examples.

Activity 3: Relating prospective teachers' explanations with their level of content awareness (engineering context)

METHODOLOGY

The participants were 86 undergraduate mechanical engineering students. The students were mostly in the 6th semester of their studies. The data are from their final exam on a Didactics course. During this course the students are introduced to teaching techniques, designing interventions and developing learning activities. They also obtain experiences in analyzing school textbooks and handling text and images for the enhancement of their students' learning. The extract given to the participants before their final exam was taken from the textbook "Car systems". The topic referred to "Helical springs as parts of the car suspension system". A part of the textbook extract given to them is presented in Fig. II.2.

The extract includes both text and visual representations. The following task was part of the participants' teaching plan:

> Describe in an analytical way how you will explain to your students the significance of the different characteristics of helical springs (linear and progressive) in controlling cars' vibrations".

Although the whole textbook extract explains the scientific part adequately (e.g., Hook's Law and linear relations), the connections with the applied context (car suspension) is absent. However, all the participants had taken the course on "Motor technology" that included topics on the car suspension system. Therefore, the participants should connect the textual information with information taken from other resources (previous courses or internet), objectify the relation between the scientific and the applied context and communicate this relation effectively to their students.

Title: Cars' suspension system; *Subtitle:* Helical springs
Main text: Presents Hooks' Law and explains the visual representations as follows: In Fig. 4.27a we see two springs with different stiffness (different constant c). The spring with c=25000N/m [is named on the graph as stiff] is considered as more stiff than the other with c=7500N/m [is named on the graph as floppy]. In the common springs, the characteristic is a straight line and is called 'linear characteristic'. If the spring stiffness increases not linearly in respect to the change of the springs' deformation (Fig. 4.27b) then we have a progressive characteristic curve".

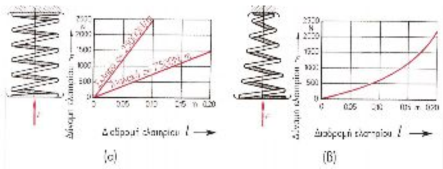


Fig.4.27: Characteristic lines of helical springs: (a) Linear; (b) Progressive

Fig. II.2: A part of the textbook extract

(Andrinos, N. Panagiotidis, P. & Papadopoulos N. 2009. Car Systems I, Athens: OEDB, p. 235).

The data analysis of all participant-produced texts helped us to place them on different levels of awareness and attention on transforming the school text into an explanatory unit.

FINDINGS

A careful study of our data led us to discern three different levels of awareness and attention in terms of the functional relation between the scientific and the applied context. The first level of awareness is expressed by participants who only presented their lesson plan without any sign of developing a deeper understanding of the text and the teaching goals. This level is called *superficial explanatory* level of awareness since it corresponds to a limited pedagogical content knowledge. Conceptual elements were mostly absent, i.e. participants did not consider how their explanations could facilitate their pupils' learning of scientific notions. The second level of awareness was expressed by participants who were able to enrich the textual material by interpreting features of the scientific context (e.g., reason on the different characteristics of the springs) or the applied context (e.g., reason on choosing the one or the other type of springs on the car suspension system). These participants, however, did not manage to create a new context where the scientific and the technical aspects are related in a functional way. The second level of awareness can be characterized as *partial explanatory*. It corresponds to a more developed degree of pedagogical content knowledge than the superficial explanatory but their responses were restricted either to the applied or the scientific context. In the third level of awareness, participants' explanations managed to create a new learning object where elements from the scientific and the applied context co-exist. Participants were able to recognize and interpret symbolic entities (e.g., Hook's Law) according to the needs of the specific situation to which they referred. Participants also appeared to be aware of their teaching goals and the subsequent actions to be taken into account in their transformations. This third level is characterized as *substantial explanatory* level of awareness. Examples of the three levels of participants' awareness are presented below. The qualitative analysis of each example follows and the results of the quantitative analysis are also discussed.

The following is an example of text falling into the superficial explanatory level of awareness:

> "I could bring in the class two springs, a stiff and a floppy. In this case, they will have the chance to observe the consequences of the oscillation in the different springs. In this way, I could relate them to the car suspension and how they (the springs) are interrelated to the car movement" (st78).

Table II.7: Analysis of st78's response (superficial explanatory)		
The role of school text		Does not use it as a tool
Participants' text		Narrative
Scientific elements	Math	Absent
	Physics	Use of scientific terminology
Applied/technical		Present
Aspects of periodicity		Implicit

In st78's response, there is not a clear goal of how he/she intends to explain the conceptual connection of experimental demonstration to car suspension and movement. Participant's attention is on an experimental activity and not on explaining how this is connected to the understanding of the real technical situation. The conceptual and pedagogical concerns seem to be superficial.

An example of text falling in the partial explanatory level of awareness is the following:

> "The two springs have a different structure, since one is cylindrical and the other conical. In the diagram (a), the spring displays a linear characteristic since it always has the same stiffness (constant c) in relation to the spring in the diagram (b) where as the length of the spring increases, its width decreases, so in the different width it has different stiffness, so the diagram is a curve" (st85).

St85 seems to be familiar with the scientific sign conventions, but not able to relate them to the referential situation (car suspension). St85's descriptions show lack of ability in relating the two contexts, the scientific and the applied.

Table II.7: Analysis of st85s' response (partial explanatory)		
The role of school text		Enriched
Participants' text		Argumentative
Scientific elements	Math	Interprets sign conventions
	Physics	Entities
Applied/technical		Absent
Aspects of periodicity		Implicitly

We found that 42.79% of the participants fall in this level of awareness (N=29). In the case of *the role of school text,* it was mostly copied or mimicked (N=17). In the case of *participants' text,* it was mostly argumentative (N=13). At this level of awareness, the participants used arguments taken mostly from the textual material and valued the scientific elements. However, they did not acknowledge the importance of the specific situation.

An example of text falling in the substantial explanatory level of awareness is the following:

> "If we want slow response on the absorbance of car' vibrations and consequently on car road holding, we choose the cylindrical spring. The reason for this choice is that its deformation (l) is related linearly to the load (Force F) imposed on it. We will not have the same effect if we choose a conical spring with a progressive characteristic since the relation between the deformation (l) and the force (F) which is imposed on it is not linear. For small loads, we will have immediate response, but for larger forces we will not have the analogous deformation and the forces will be transferred to the passengers' cabin" (st36).

Table II.8: Analysis of st36's response (substantial explanatory)		
The role of school text		Enriched
Participants' text		Argumentative
Scientific elements	Math	Interprets sign conventions
	Physics	Connect physical entities & Laws
Applied/technical		Present
Aspects of periodicity		Implicit

St36 starts by making an argument, namely that cylindrical springs have slow response on car vibrations. Moreover, St36 relates the car vibrations to the holding capacity of the car and uses the linear relation as warrant for slow car response on vibrations. St36 interprets symbolic mathematical relations and connects the physical entities with the physical laws. It appears that st36 uses a developed structure of attention and is able to produce a substantial explanation for his/her own students. The participants that belonged to this level of awareness were N=13 or 19.1%. They all enriched the textual information and used arguments in their responses.

CONCLUDING REMARKS on PART IIa

The analysis of undergraduate students' responses on the three research activities used raised four main issues. The first issue is related to *students' misconceptions on the periodical or non-periodical behaviour* of a graphical representation the function $f(x)=e^{-bx}\sin(\omega x)$. The second issue raises two opposed students' cognitive attitudes i.e. *they skip justifications on their claims* while they *join in visualization practices*. The third issue concerns the existed *difficulty that students face in integrating mathematical and physics knowledge* in order to provide complete explanations in terms of topics of periodicity. Finally, *positive links between robust understanding of aspects of periodicity and the production of sufficient explanations* have been identified.

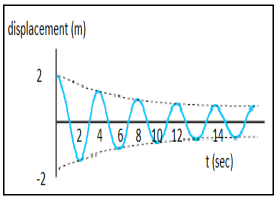


Fig. II. 3: A typical textbook image of a degreasing oscillation.

The vast majority of students easily identified the periodical property in periodic non-sinusoidal graphs. It is interesting though that a Graph, which exhibits a fluctuation (looks like the sinusoidal curve but with decreasing amplitude) (see Fig. II.3) seemed to confuse students a lot, since seven out of ten considered that it represents a periodic motion. Students' overgeneralizations, such as any repeated function is periodical, is also highlighted in Buendia and Cordero study (2005). This students' misunderstanding can be explained since in physics texts functions, such as $f(x)=e^{-bx}\sin(\omega x)$, are considered as functions that model 'periodic motions' under certain conditions. Besides, the existence of the sinusoidal function (a typical example of periodical function in mathematics and physics) adds to students' misconceptions. We have to mention that sinusoidal graphs are the main graphical models of the behaviour of periodic motions in school textbooks (see conclusions on PART I), but still are causing students false impressions.

Argumentation and reasoning seems to be a non-familiar practice for undergraduate students. The students' low participation in relative tasks, where justification was necessary, supports our claim. In the case of participating in this type of practice mostly logical-empirical type of justifications were identified in their responses. Participants rarely used formal definitions as warrants in their responses. Even mathematics undergraduate students avoid using the formal mathematical definition for periodic functions. One reason for students' reluctance to argue on their claims could be either because argumentative-pedagogy is not a common practice in the Greek educational system or because certain conceptual elements are not consciously comprehensible by them.

Our findings provide evidence on students' strong willingness to assign meaning to abstract mathematical entities. This outcome was proved both by their high participation in the 'exemplifications task' and by the fact that they use these examples as warrants for their justifications (we name this justification as Logical-empirical, general-specific reasoning practice) (Act.2). In this case, the role of students' kinesthetic experiences has been proved central both when they provided enriched examples of motions represented by the particular graphs and when they take the stance to change the context of the examples according to their perception of the graphical features represented. These findings show the embodied nature of thinking and the genetic relationship between the sensual and the conceptual in knowledge formation (Nunez, 2007; Radford, Cerulli, Demers, & Guzmán, 2004). We add more to this conception by highlighting the role of visualization in reasoning. Visualization seems to be an important skill in order to identify the periodical behaviour when making sense out of VRs. But this skill is not enough in order to overcome 'reasoning gaps' in texts. In this case, the readers need to argue by synthesizing all the logical acts (from sensual to abstract) in order to overcome diversities and reasoning gaps. This means that fragmented knowledge consisted only of nomological modes of reasoning (i.e., definitions, Laws etc.) or only empirical or logical-empirical modes of reasoning are not adequate in order to develop a sufficient argument on complicated tasks (e.g., in Act. 1 when students were asked to explain the negative values of F in Fig. II.4).

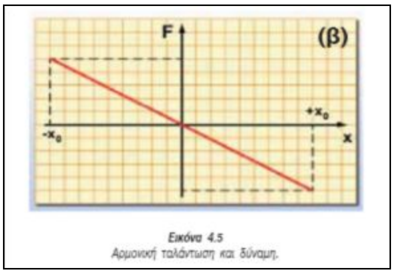


Fig. II.4: The variation of restoring Force in Simple harmonic oscillations.

An important demand of the task for undergraduate students was to integrate their mathematical and their physics knowledge in order to provide sufficient explanations concerning texts on periodicity (Act 1 & Act. 3). Three dimensions capture how undergraduate engineering students (future teachers) transformed a text on car suspension into an explanatory teaching unit: the role of the school text; the type of text produced by the participants; and the conceptual elements identified in participants' texts. The last dimension provided evidence to discern three levels of participants' awareness of the functional relation between the scientific and the applied context: the *superficial*, the *partial* and the *substantial explanatory*. Qualitatively speaking, explanations became more sophisticated when participants fall in a higher level of awareness.. Particularly, at the second and the third level of awareness, participants employed and enriched the information in the textbook extract, while their text became less narrative or expository and more argumentative. Moreover, in the *substantial explanatory* level participants were able to express connections between different fields of knowledge in an argumentative way.

The complexity of integrating contextual and scientific salient elements when transforming a scientific text is apparent in this study, since only a small number of participants managed to reach the *substantial explanatory level of awareness*. Finally, issues on the value of explanations in teaching have been acknowledged (Perkins & Grotzer, 2005) and their relationship to conceptual aspects remains an issue worthy of further research (Braaten & Windschitl, 2011).

PART IIb

Teachers' pedagogical tools when teaching periodicity

METHODOLOGY

The participants

A total of 50 teachers participated in this research phase. 42 and 13 participated in research activities 1 and 2 respectively, while 5 of them participated in both activities.

Table II.9: Participants in PART IIb.

Research activity	Engineering	Mathematics	Physics	TOTAL
1	5	19	18	42
2	3	5	5	13
TOTAL	8	24	23	55

The data analysis was based on the grounded theory research perspective (Corbin & Strauss, 2007). In particular, we are looking for categories and patterns emerging from the analysis of the raw data. More specifically, inductive content analysis (Mayring, 2000) was applied and a coding system of categories has been produced. In some cases our analytical framework on MsoR (Triantafillou, Spiliotoulou & Potari, 2015) provide us a set of filters through which to systematically examine teachers' justifications (Act.1) and if and how they modify the argumentation developed specific thematic units on periodicity (Act.2).

We have to mention teachers' reluctance to participate in this study and answer in all the tasks.

Activity 1: "Teachers' justifications and fundamental images on periodicity" (questionnaires)

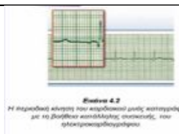
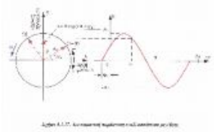
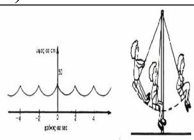
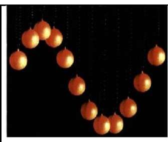
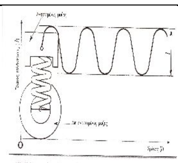
A questionnaire with open-end questions was either distributed as hard copy to some educators in 13 schools in three Greek cities, or sent electronically to others. In this activity our focus was on images of the notion that we met in school texts and on teachers' justifications when they had to eliminate certain students' misconceptions about the notion.

Task 1

We ask the educators to refer to the basic teaching units they teach that contain aspects of the notion of periodicity, choose one teaching unit of the above and make a list of the fundamental visual representations (VRs) of the notion that they meet in the school textbook in the specific teaching unit.

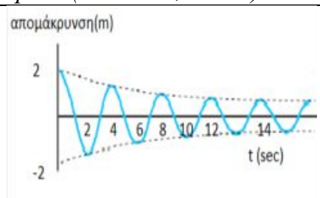
Task 2

In this task, we question the actual or potential use of five VRs selected from five different school texts. We also asked them to indicate their teaching goals (in the cases of actual and/or potential use of specific VRs).

<table>
<tr><td colspan="5">Table II.10: The five images of periodicity (VRs)</td></tr>
<tr>
<td>
VR1</td>
<td>
VR2</td>
<td>
VR3</td>
<td>
VR4</td>
<td>
VR5</td>
</tr>
</table>

Task 3

We use results taken from PART 1a. In the following task, we asked the participants to describe in brief how they could handle two characteristic students' responses (in cases when these came up in their classes).

<table>
<tr><td colspan="2">Students' Task</td></tr>
<tr><td>Graph 4 (PART 1a, Act. 2)</td><td>Questions</td></tr>
<tr><td>
</td><td>Does this graph represent a periodic motion? Justify your answer.</td></tr>
<tr><td colspan="2">Students' responses</td></tr>
<tr><td colspan="2">R1: «It is periodic because it represents the motion of the swing» (we refer as R1-swing)</td></tr>
<tr><td colspan="2">R2: «It is periodic because every sinusoidal function is periodic» (we refer as R2-sinx)</td></tr>
</table>

Teachers' task: Describe in brief how you could handle the above students' responses in cases these came up in your class.

Graph 4 was a graph that caused a lot of misconceptions to students. We chose the above students' responses because they all refer to the sinusoidal function either explicitly (R2) or implicitly (R1). Moreover, the motion of playground swing is a typical example of periodic motion in mathematics and physics (see findings on task1 in this activity). Besides, theses responses were typical and seemed to predominate students' conception that a function that fluctuates about the x-axis with decreasing amplitude (i.e. $e^{-x}\sin x$) is periodic and is represented by the sinusoidal function. Teachers are asked to make an argument in order to eliminate students' misconceptions.

FINDINGS

Task 1

The basic thematic units and fundamental visual tools teachers referred are presented in Table II.11

Table II.11: The fundamental images of periodicity in school texts*

Educators' subject	*Teaching unit*	*Fundamental visual representations in school texts*
Engineering	alternate currents	generator of alternate current and the corresponding sinusoidal function, photo of moto-electrical devices
Mathematics	trigonometry	sinusoidal graphs, other graphs of trigonometric functions, the trigonometric circle, drawing of playground swing
Physics	Harmonic oscillations	Sinusoidal graphs, graphs of linear relations, the photo of the pendulum clock, photo or drawings of the playground swing, schematic representations of experiments with springs and simple pendulums

*Some of the educators added the use of digital technology (videos, animations or simulations).

According to our data, the most fundamental images of the notion in school texts are:

(a) The *sinusoidal curve* which seems to dominate educators' practices in all subjects. The educators use this curve in different contexts and for various purposes (i.e. in the symbolic form of $I = I_o\sin(\omega t)$ to study alternate currents in engineering courses, in the symbolic form of $y = \sin x$ to study trigonometric functions in mathematics and in the symbolic form of $y = A\sin(\omega t)$) to study simple harmonic oscillations in physics.

(b) The *pendulum swing*. This image appears in different genres and in different contexts and seems to be a common image of the notion in mathematics and physics texts. Particularly, the swing of the pendulum image appears as a photo of a pendulum clock in physics or as a drawing of a playground swing in physics and mathematics or as a schematic representation of a simple pendulum swing (a pendulum swing consists of a relatively massive object hung by a string from a fixed support) in physics.

(c) The *trigonometric circle* is a fundamental image of the notion in mathematics.

Many educators from all subjects consider that the role of a visual representation in a text could be either to attract students' attention or to visualize the real situation. Educators in engineering courses consider as important synthetic images consisting of different types of VRs that represent periodical variations. As an example, we provide the following image that is considered as important by Q25_eng participant. This image is a synthesis of a graph (the sinusoidal curve), a drawing (generator of an alternate current) and two photos (the alternator). This synthetic image role in an engineering text (Car electrical system) is to explain the operation of this moto-electrical device (in this case, the alternator). This image incorporates elements of mathematics, physics and technology.

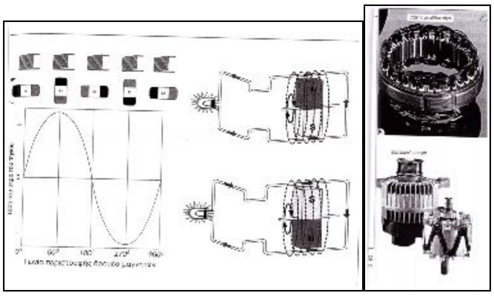


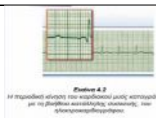
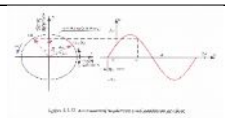
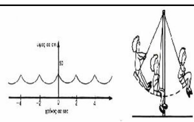
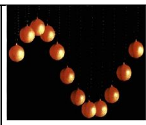
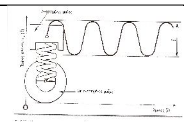
Fig. II.5: The VRs' mentioned by an engineering educator.

This indicates that engineering is a complicated context where interpreting images with different levels of generality is considered to be a fundamental skill.

Task 2

The most preferable VR by all educators was VR3 (the synthesis of the drawing of a playground swing and the periodic graph). The second most preferable VR by all educators was VR2, in the third place we meet VR4. The respondents put VR1 in fourth place and VR5 last. We note that teachers could use more than one VR.

Table II. 12: The five images of periodicity (VRs) and teachers' hierarchical preference of the five VRs. (Total No of participants in this Task 38)

 VR1	 VR2	 VR3	 VR4	 VR5
50% (N= 19)	71.05% (N= 27)	84% (N= 32)	60.52% (N= 23)	23.68% (N= 9)

The results indicate that the image of a playground swing and a periodic graph is the most preferable VR (84%). This result adds that the 'pendulum swing' is a prototypical image of periodicity. Some math educators admitted, though, that their reference to this image is very brief due to time restrictions. Some physics educators suggested changing parts of this image for potential use in their class. The suggested changes by two participants were to put the starting point of the motion at (0,0) while another three acknowledged that the period of the motion is different in physics, and this must be mentioned to students. This critical perception of an image indicates that educators many times are able to change something given in their text in order to use it as a tool in their teaching practice.

The second most preferable image was VR2 that represents the clockwise circular motion of α vector representing the alternate current $a(t) = Ao \sin(\omega t+\varphi)$ (71.05%). The angular velocity of the above motion is ω. The starting point of the motion (t=0) was ($A_o \sin(\varphi_o)$). Next to this motion is its graphical representation. This image layer of generality is considered abstract, as no physical situation is represented while a variety of mathematical objects are present (the trigonometric circle, the vector representations of the alternate current), the sinusoidal graph as the basic model of periodic variation and many symbolic entities ($a=A_o \sin(\omega t+\varphi)$). This image's potential use seems to dominate physics and engineering teachers' practices, while only a few math educators mentioned its potential use in their class. The issue that this image reasons visually on the generation of the sinusoidal function, but in the context of alternate current, seems to be restricted by most of the math educators (maybe they were not familiar with many symbolic entities in this VR).

In the third place we find VR4 (60.52%). It is an elaborated photo (chronophotography) and its layer of generality is concrete. VR4 represents consecutive instances of the oscillating sphere attached on an ideal spring. In the fourth place we find VR1 (50%) that represents the motion of the cardiovascular muscle.

It seems that the level of generality is not always teachers' criterion for using an image in their teaching practice but the content and the context of the image. Particularly, their preference seems to depend on (a) how central they consider it in their teaching and (b) how they could handle it in the class. It is interesting that many mathematics educators suggested using VR4 (an elaborated photo image) in their class although in Greek mathematics texts the presence of photos is almost rare. Also, it is interesting that although in physics texts the only periodic graph is the sinusoidal curve they value other types of curves that exhibit a periodic behaviour not present in their text as important in their teaching as well.

The teaching goals mentioned by mathematics and physics educators are presented in Fig. II.6. Mathematics and physics educators appear to have almost the same teaching goals when using the above VRs. Particularly, modelling is considered by them as their prior teaching goal (either by studying the mathematical model or by relating the phenomena to their mathematical models). Moreover, 'making connections to every day phenomena' is valued high even for mathematics teachers and this result indicates that teachers' need to find images that could help them to make connections to every day life. If we examine the teaching goal for each VR separately, we see that each educator have different learning objectives even in the same subject. Since different learning objectives could produce different teaching practices, we conclude that the use of the same image in school practice does not necessarily presuppose the same activity.

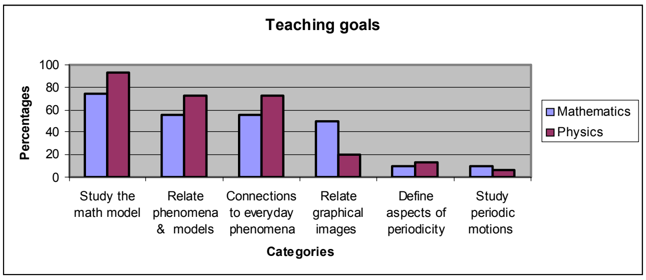
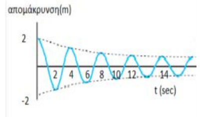


Fig. II.6: Mathematics and physics educators' teaching goals.

Task 3

In this task we asked the educators to describe in brief how they could handle two characteristic students' justifications (R1-swing & R2-sinx).

<table>
<tr><th colspan="2">Students' Task</th></tr>
<tr><td>Graph 4 (PART 1a, Act. 2)</td><td>Questions</td></tr>
<tr><td>
</td><td>Does this graph represent a periodic motion? Justify your answer.</td></tr>
<tr><td colspan="2">Students' responses</td></tr>
<tr><td colspan="2">R1: «It is periodic because it represents the motion of the swing» (we refer as R1-swing)</td></tr>
<tr><td colspan="2">R2: «It is periodic because every sinusoidal function is periodic» (we refer as R2-sinx)</td></tr>
<tr><td colspan="2">Teachers' task: Describe in brief how you could handle the above students' responses in cases these came up in your class.</td></tr>
</table>

We identified the following categories in teachers' responses:

Teachers suggest that they would ask students to …

… compare this particular example (e.g., the playground swing) with other examples that exhibit a periodical behaviour. Some characteristic responses are: *'I could provide the example of the pendulum clock and I could ask them to compare the two motions.* (Q18_eng); "*I would ask students to compare the every day examples with their models that we use in our classrooms"* (Q15_phys.). These responses were mostly met in the case of R1-swing.

… focus on the graph (mathematical model) and relate it with other mathematical models: Some characteristic answers are*:* '*In the case of the swing we have frictions, so as the time goes on, the maximum and the minimum of the graph changes*, *as a result it is not a periodic motion*.' (Q2_math); *'I would discuss with the students the relation of the phenomenon as it happens in nature with their mathematical models* (Q15_physics); '*I would present the sinusoidal curve and I would discuss with my students the similarities and the differences with the two curves (*Q8_ math).

In the above categories teachers create a context and ask the students reflect on it (the context could have either a physical or a mathematical situation). We could consider the above categories as Logical-empirical MsoR. The definition of periodic motions (or functions) is implicit in teachers' responses.

… use definitions (either by applying the definition or by referring to taxonomies of periodic motions). *"I would ask them to apply the definition"* (Q5_math). In these responses the context is absent. We could name these as context-free responses.

Finally, no justification is the case that teachers are judging students' responses as correct or incorrect with no further comments e.g., "*we can not define the notion periodicity through an example*" (Q7_math); *The answer is not right because* $f(x+T) \neq f(x)$(Q6_math).

In the following table we present the results of our quantitative analysis for all participants. It seems that the context of students' justification affected teachers' responses. In the case of (R1-swing) teachers preferred to ask students to compare concrete examples of periodic with examples that exhibit periodical behaviour (38.5%). In the case of R2 (sinx) most teachers (56.5%) preferred to ask students to focus on properties on the mathematical model. It is interesting that the vast majority of teachers set up a context and proposes logical-empirical modes of reasoning in order to clarify students' misunderstandings in both Rs (almost 70% for R1-swing & almost 60 % for R2-sinx). It is surprising though that 15% to 17% of the teachers provide the answer with no justification. We do not know if this is a common practice in their class or this was a way to avoid making an argument.

Table II.13: Results of quantitative analysis for all participants.		
Categories Asking students to …	R1 (swing) N=26 (%)	R2 (sinx) N=23 (%)
… compare this particular example (e.g., the playground swing) with other examples that exhibit a periodical behaviour	38.5	4.34
… focus on the graph (mathematical model) and relate it other math. models	30.76	56.52
… apply definitions	15.38	22
Provide the answer with no justification	15.38	17.4

In the following table we focus on mathematics and physics educators responses by taking into consideration their responses in both tasks (R1-swing & R2-sinx).

Table II.14: Results of quantitative analysis in both R's in the case of physics and mathematics teachers		
Categories Asking students to …	Math. N=24 (%)	Physics N=21 (%)
… compare this particular example (e.g., the playground swing) with other examples that exhibit a periodical behaviour	16.66	28.6
… focus on the graph (mathematical model) and relate it other math. models	41.66	38.10
… apply definitions	25	14.28
Provide the answer with no justification	16.66	19.05

Even though physics teachers prefer to use examples (set up a context in their justification) while math teachers prefer to use definitions (context-free justifications) we can speak about common reasoning behaviours between them.

Activity 2: "Teachers' judgments when modifying textbook argumentation" (interviews)

The interview theme in this research activity was based on the role of the argumentation developed in textbooks when topics on aspects of periodicity are presented in teachers' classroom practices.

We focus our attention on different topics according to each participant: For mathematics teachers the topics are: Define periodic functions & Study the sinusoidal function (Mathematics, 2nd year in upper secondary school). For physics teachers: Define periodic motions (Physics, 3rd year in lower secondary school) and Define simple harmonic oscillation (Physics, 2nd year in upper secondary school). The teachers in engineering courses choose the topic themselves.

We adopt the position that textbooks aim to introduce their readers to the conceptual aspects of scientific and mathematical knowledge and persuade them of their value. In this way we consider that a form of argumentation is developed in each thematic unit and is produced by a sequence of modes of reasoning (MoR) that the author develops in the text when organizing and presenting the new knowledge. We ask teachers if they follow these MsoR or they modify them, how and why?

Some questions addressed to teachers were: *In the school text the new knowledge is developed in a certain way. Do you follow this when you teach one of the above topics*? *Are there some parts on the development of the new knowledge that you pay more attention to when you teach this topic*? *If yes, which ones exactly*? *Do you use in your teaching practice the examples provided in the textbooks or other examples? Can you specify? How do you connect the examples with the topic you teach? Please justify your answer.*

All the participants' names appear below are pseudonyms.

FINDINGS

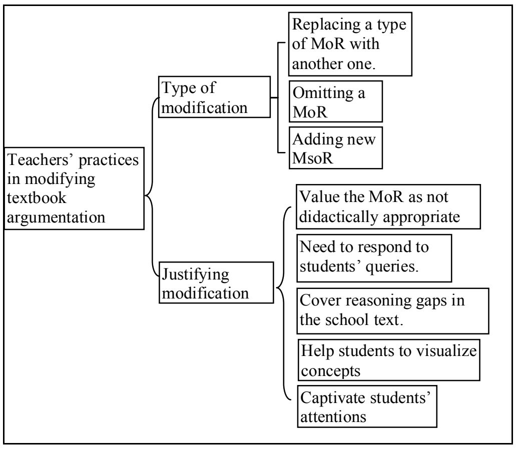


Fig. II.7. The systemic network (Bliss et al., 1983) on teachers' justification on their text modifications.

Most of the participants express many concerns about the textbooks *e.g. "I think that the books say many things but not purposely ... they tell a story but nothing in particular"* (Andreas, eng.). As a result, most participants in this activity mention that they prefer to modify parts of the knowledge organization provided in each thematic unit.

Teachers' practices in modifying textbook argumentation emerged from the data analysis in a form of a systemic network (Fig. II.7). Two main dimensions characterize teachers' practices: The *type of modification* (replacing or omitting or adding a MoR) and their *justification*.

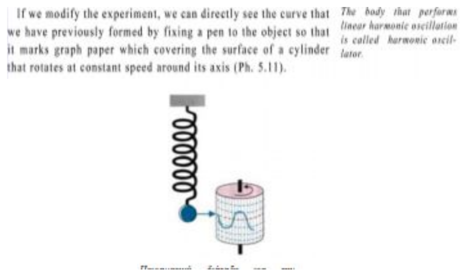
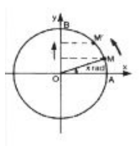


Fig. II.8. The textbook Logical-empirical MoR that Vaggelis (physics) replaces with an Empirical one.

Their justifications in modifying textbook reasoning were as follows (i) *when valuing the MoR as not didactically appropriate*; (ii) *when responding to students' queries; (iii) when covering reasoning gaps* in the school text; (iv) *help students to visualize concepts*; and (v) in order to *captivate their students' attention*.



[The school text] We notice that as x values from 0 to π/2, point M moves from A to B. Therefore, the y-coordinate increases, thus the function sinx is strictly increasing in the interval [0, π/2]. Similarly, we find that the function is strictly decreasing in the interval [π/2, π]. [...] Moreover, the function has a maximum value on x=π/2 (sinx=1) and a minimum value on x=3π/2 (sinx=-1).

Fig. II.9: Example of an omitting textbook Logical- empirical MoR.

We exemplify our analysis below:

In the following extract Vaggelis (physics) replaces a logical - empirical MoR (see Fig. II.8) to an empirical one because he values that the context of textbook MoR is not appropriate (the experiment is impossible to be successful):

Vaggelis (physics): *I never use this example [...] (Fig. II.8) I think that this experiment is impossible to be successful.*

Besides, my students can do this experiment by themselves; one student holds a pen in his hand and moves his hand vertically in a constant way while his colleague moves a paper before him with a constant speed. The pen sketches the sinusoidal curve on the paper.

The case of omitting a MoR was mentioned by participants from all subjects.

For example, Niki (math) mentioned that usually she does not use the trigonometric circle in order to study the sinusoidal function (Fig. II.9) and argues as follows:

Niki (math): students do not realize easily that the sinusoidal function has period 2π. They ask me 'what is π?'. So, I start reminding them how we defined 'π' in geometry. So, after omitting the part with the trigonometric circle, I could go straight to the value table that they know and I use that to sketch the sinx graph.

In this case the math teacher omits a central MoR (defining the sinx function with the help of the trigonometric circle) since she has to remind to his students a lot of theoretical issues (e.g. define 'π'). In this case Niki (math) omits a central Logical empirical MoR and adds other Nomological ones (e.g., defining 'π') in order to respond to students queries. These MsoR are parts of previous thematic units.

On the other side, most physics teachers mention that they try to cover the reasoning gaps in textbooks in the case of identifying the sinusoidal function in by using the trigonometric circle. For example, Fani (Physics) mentions:

"when I teach the Simple harmonic oscillation I have to define the sinusoidal function, for this reason I sketch the trigonometric circle and take certain values, T, T/4. T/2 etc. ... this is the only way for the students to follow my lesson".

In this case, an omitting MoR by a math teacher (Niki), due to other didactical issues that came up in her class, is an adding MoR by a physics teacher (Fani) in order to cover reasoning gaps in physics textbook.

In the following two extracts Nikos and Ntinos mention that they usually omit a particular example when teaching the thematic unit 'Periodic motions', 3rd grade lower secondary school (Fig. II.10).

The school text: [...] The muscle of the heart performs a periodic motion as well, as represented in the electrocardiogram [photo 4.2]

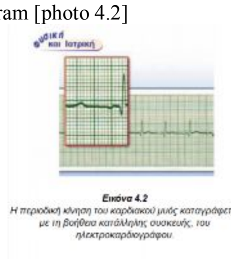


Fig. II.10: An example of an omitted Logical-empirical, general-specific MoR.

Nikos (physics): *"I never use the electrocardiogram as an example of a periodic motion, its is a very strange example, since the conditions should be perfect in order to have such a diagram".*

Ntinos (physics): *"I prefer not to use the example with the electrocardiogram since it is hard for me to explain what parts of this graph are represented".*

The reason for this change in the knowledge organization developed in the textbook is because they valuing the MoR as not didactically appropriate.

Giannis (eng.) changes a sinusoidal image as follows when he teaches "Alternate currents" to his students:

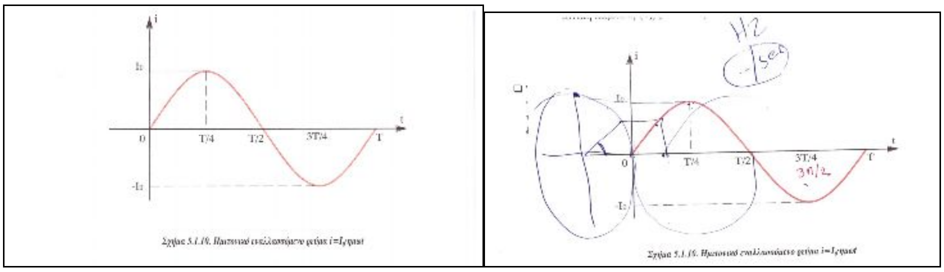


Fig. II.11: The textbook VR (appears as a Nomological MoR-definition of Altenate Cuurent- and the way Giannis (eng.) changes it in order to make his students understand what “c/s or Hertz” means:

> "*We have the sinusoidal curve which shows [the values] 0 - max - 0 - min and I tell them if this part of the curve [T/2-T] flip it under the first part it makes a cycle ... that is why we refer to the cycle per second which is the Hertz ...* ".

Giannis (eng.) adds a Logical-empirical MoR that was created by himself in order to make clear to his students what “c/s” means. The reason for this modification is to help students visualize a central notion in their profession.

Two physics participants mentioned that instead of proving the formula $T=2\pi l/g$ (the proof is in 3rd grade upper secondary school physics textbook- a Mathematical MoR), they prefer to provide the formula and use the interactive physics software in order to ‘verify’ that the period of the oscillation is related to the above quantities in the way represented in the formula. In this way they change a mathematical MoR to a logical-empirical one (we consider that an argument that was made by digital technology is a Logical-empirical MoR). They mention that in the verification process the students were more involved than in the proof practice and they are "*more convinced of its truth*". As Ntinos (physics) mentions "*For many years the students thought that the formula is something coming out of the blue, now the digital technology could help students to realize that this is not the case"*. According to Ntinos this is very important since "*it influences students' understanding*" and make didactically appropriate arguments on verifying science Laws. Many participants mention adding arguments (in our study MsoR) in order to help their students visualize main concepts in their profession.

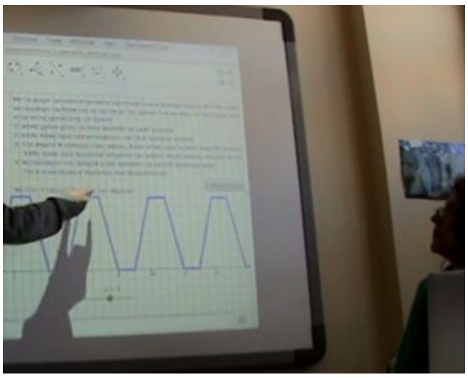


Fig. II.13: The textbook generic example becomes ‘alive’ with the help of digital technologies.

The use of animations of periodic motions as teaching tools (videos and educational softwares e.g. interactive physics; Geogebra) is mentioned by six teachers from the 13 participants. For example, Vaggelis (physics) mentioned that sometimes in his class uses videos showing periodic macrocosmic and microcosmic phenomena such as *"the oscillation of the molecules in solids*" (see Fig. II.12). Mostly this

Fig. II.12: An case of adding a MoR in order to captivate students’ attention.

happens in the introduction of the thematic in order to captivate his students' attention. Magda (math) uses an animation (Geogebra) in order to make her students visualize better the textbook generic example (see Fig. II. 13). In this case, the textbook generic example becomes 'alive'.

All teachers when introducing the notion of periodicity mention that they use every day examples not included in their textbooks. These examples could be "*the motion of the pendulum clock*" or *"the weekly publication of magazines*" or natural phenomena taken such as the *"the day-night shift"* or "*the phenomenon of the tides"*. The use of the periodic example of the *"menstrual cycle"* was mentioned by three teachers.

The reason for this enrichment is because they want to captivate their students' attention. In most cases though this MoR (recalling students' experiences with periodic phenomena) seems to be disconnected from the textbook argumentation. Sometimes this teachers' attitude is in conflict with the use of examples in textbooks. Examples in school texts seem to play a significant role in the inherent logic of new knowledge development. Particularly, they are either the basis for generalization (generic examples) or the applications of general statement.

"Teachers suggestions" (Common Task in Act 1 & Act 2)

In this task we asked teachers to write down their suggestions on how they could help their students to develop a unified view on periodicity where aspects of the notion from the different subjects coexist harmonically.

FINDINGS

The following categories emerged from the analysis of participants responses:

- provide many examples of everyday life periodic phenomena *(e.g. the periodic motion of the earth around the sun*)
- define & explain carefully the mathematical models
- Help the students make links between the concrete situations and the mathematical objects that they model them (e.g., use animations of periodic motions)
- propose the use of inter disciplinary projects and co-operate with educators from other subjects

The quantitative analysis of the 26 teachers who responded in this task we take the following results. We have to mention that some teachers made more than one suggestion.

TABLE II.15: Teachers' suggestions

Categories	*Frequency N=26 (%)*
Provide many examples of everyday life periodic phenomena *(e.g. the periodic motion of the earth around the sun*).	46%
Help students to make links of the concrete situations with the mathematical objects that they model them (e.g., by using animations of period motions).	39%
Define and explain carefully the mathematical models	38%
Propose the use of inter disciplinary projects and co-operate with teachers from other subjects.	12%

In first place was the suggestion: '*provide many examples of everyday life periodic phenomena*'. Teachers from all subjects proposed this category. In second place was the suggestion on “help students make links of the concrete situations with the mathematical objects that they model them (e.g., by using animations of period motions). The last category was ‘*use of inter disciplinary projects and co-operate with teachers from other subjects’* was proposed by only a few participants.

We have to add teachers’ ignorance of texts from other subjects. Physics teachers never had the chance to see how mathematics textbooks introduce the students to the notion or periodic function, nor how they address the sinusoidal curve. The same applies to the mathematics teachers.

CONCLUDING REMARKS on PART IIb

50 secondary teachers from the subjects of engineering, mathematics and physics participated in this part of our study. The analysis of teachers’ responses on the two research activities raised four main issues: The first issue is that the *sinusoidal curve is qualified as the prototypical image of the notion across subjects*. The second issue is teachers’ willingness *to enrich their teaching with every day images and examples of periodic phenomena*. The third issue is that *although mathematics and physics teachers share common learning objectives, images of periodicity and reasoning practices they avoid co-operating* on didactical issues. The last issue concerns *teachers’ preference to modify the textbook argumentation*. It appears that teachers’ modifications might unconsciously influence severely the inherent logic of the concept presentation in classrooms.

The sinusoidal curve is qualified as the prototypical image of the notion across subjects, while the sinusoidal function seems to be a common teaching tool for engineering, mathematics and physics educators. Mathematics educators mention using sinusoidal functions, either to solve trigonometric equations, or to study their characteristics (e.g., period, maximum values, etc.), while physics and engineering educators mention using this function to model the periodical change of a number of quantities in the course of time (Act 1/ task 1).

All participants mention a lot of examples used in their lesson when teaching aspects of periodicity. These examples vary from phenomena close to students’ experiences (menstrual cycle) to natural phenomena taken from the microcosm (the vibrations of the particles) and the macrocosm (the motion of the planets) (Act 2). Besides, many teachers suggest that making connections to everyday life periodic phenomena could hep students to develop a unified view of periodicity. From the analysis of our data in Act 2, it seems that teachers use every day phenomena mostly in order to stimulate their students' attention. This teachers’ attitude might be in conflict with the use of examples in textbooks. Examples in textbooks were used purposely either to make generalizations (generic examples) or to provide applications of the notion presented in the thematic unit.

In general, all participants were critical of the textbooks used in their classroom teaching. Consequently, they prefer to modify parts of the new knowledge organization provided in each thematic unit. The modifications mentioned by the teachers could be omitting or adding or replacing the modes of reasoning presented in the textbooks. These modifications are taken place when (i) they value the MoR as not didactically appropriate; (ii) they aim to respond to students’ queries; (iii) they cover reasoning gaps in the school text; (iv) they aim to help students to visualize concepts; and (v) they want to captivate their students’ attention.

Mathematics and physics teachers share common notions (e.g. periodicity); common images (e.g. the sinusoidal curves); mostly common reasoning behaviours when handling specific students' responses and learning goals (e.g. modelling). Despite all these commonalities it seems that co-operation with their colleagues in neighbouring subjects in planning and reflecting on their teaching practices is not a preferable attitude.

Ch. 4

CONCLUSIONS of our STUDY

Learning and understanding periodicity involves reasoning about its properties and characteristics, synthesizing aspects of the notion from different contexts and epistemological fields and be able to apply them in a repertoire of reference practices which include and the transfer of this knowledge in new settings.

In this study we investigated textbooks' characteristics and teachers' pedagogical practices in terms of reasoning and argumentation, as these may be related to students' learning and understanding periodicity. From textbook analysis, the crucial role of argumentation in conceptualizing periodicity emerged. Subsequently, we investigated how secondary teachers enact on the textbook argumentation and we tried to identify how these two factors, textbooks and teachers' practices could contribute or limit students' conceptualization of the notion.

The synthesis of our results provides us with evidence on the following issues:

- *Productive links between textbook argumentation and conceptualization:*

Argumentation in the 'content presentation sections' of a textbook is made through the series of MsoR (parts of the thematic unit that state a syllogism crucial for the development of argumentation in the whole), the VR's genre and the co-deployment of the MsoR and VRs. In terms of MsoR, the empirical and logical-empirical MsoR help the reader to make links between their sensory perceptions of the notion and its general characteristics. The nomological MsoR indicate the epistemological and ontological aspects of periodicity aimed to be learnt in each subject. Finally, the mathematical MsoR provide support to abstract ways of thinking where representations of periodicity are mostly symbolic in nature. In terms of VRs' genre, the photos and naturalistic drawings of periodic motions help readers to visualize everyday periodic phenomena while the schematic representations and the graphs mostly support abstract ways of thinking about the notion. We note that the absence of particular MsoR and VRs or the functional connection of a VR with a MoR could result in a text where conceptual elements of periodicity, or links that are important for understanding are missed. Hence, we argue that inside a school text the deployment of argumentation and conceptualization is inevitable, while understanding through reading is viewed as occurring through the dialectical relationship between these two channels of thought.

- *Pragmatic considerations on readers' understanding of the text argumentation and how teachers might enact on this practice*:

The analysis of textbooks argumentation indicated that reasoning gaps in texts could influence understanding of the argumentation developed in the text (e.g. the sinusoidal curve comes arbitrary in a physics text). Teacher might enact on this practice by modifying the sequence of MsoR and hence influence the flow of argumentation when teaching these specific thematic units. Teachers' modification could be made for several reasons (e.g., when valuing that a MoR is didactically inappropriate or in order to stimulate their students' attention). Besides, mathematics, physics and engineering teachers mention that they enrich their teaching with every day images and examples of periodic phenomena, besides the ones provided in the text. We argue that teachers' appropriate modifications might sustain students' visualization of periodic motions (e.g., by supporting logical-empirical MsoR with the use of digital tools) or add to their understanding of text argumentation by covering reasoning gaps in texts (e.g., physics teachers involve the trigonometric circle in order to reason about the sinusoidal curve). In this case teachers' modifications in most cases enact constructively in students' experiences with text argumentation. Besides, there are some cases that the teacher unconsciously might limit the inherent logic of the concept presentation of the text, either by omitting specific MsoR that are important in the argumentation developed in a thematic unit, or by not placing the every day examples of periodic phenomena as integral parts of the argumentation developed in the thematic unit. This fact highlights the importance teachers, on one side to appreciate all syllogisms from sensory perceptions to abstract thinking and reasoning as important rational actions in concept formation, and on the other side the need to make transparent to their students the distinction between the different roles of the above range of syllogisms in the development of a sound argument. The fact, that students are not on their own and naturally involved in reasoning and justification practices, makes the above issue a teaching necessity.

- *An attempt to explain why the main image of periodicity in texts and in teachers' practices (the sinusoidal curve) still is a source of students' misconceptions*

VRs as images of periodicity play a major role in the inherent logic of the concept presentation in texts. The main images of periodicity are the sinusoidal curve and the trigonometric circle. These two images are linked, since the sinusoidal curve comes as a result of a series of MsoR that are based mostly on the trigonometric circle (a reasoning practice common in math, physics and engineering educational community). The sinusoidal curve is acknowledged by all teachers as their main teaching tool, while 20% of images in all school texts analyzed are sinusoidal graphs. For all these reasons we consider the sinusoidal curve as the prototypical image of periodicity across subjects. This comes in conflict with undergraduate students' misconceptions on the sinusoidal curve. Particularly, students consider that the sinusoidal curve (f(x)=sinx) shares common properties (here the periodical behaviour) with the function $f(x) = e^{-bx}\sin(x)$. Possible explanations of this students' misunderstanding could be: (a) In physics texts, the distinction of the two functions in terms of their periodical behaviour is not so clear; (b) the absence of non-examples of periodic functions in all texts; (c) students miss parts of the argumentation, where the sinusoidal curve was defined (i.e. if readers do not pass through certain MsoR resulting in the sinusoidal curve then important conceptual or logical elements may be missing). Students' overgeneralizations such as that any repeated function is periodical are also highlighted in Buendia and Cordero's study (2005).

It is common practice in Mathematics textbooks to avoid using photographs (that represent particular instances) and the use of images that convey a generality is prefered (Herbel-Eisenmann & Wagner, 2007). We argue that this practice is purposeful, since authors seem to believe that these images do not support a 'proper mathematical' argument. Although recent research has stressed the decisive and prominent role of bodily actions and gestures in students' elaboration of elementary, as well as abstract mathematical knowledge (Núñez 2000), this is neglected by Greek math textbooks' authors. On the other side, physics textbooks try to bring the notion closer to readers' empirical experiences in the 'content presentation sections', although this practice is not used also in the reasoning practices required in the ‘textbook proposed exercises’. Most of the exercises in physics and almost all the exercises in mathematics texts on periodicity are context-free. The above practices could narrow students’ enrolment in understanding the different aspects of the notion (e.g., modelling activities with real-life periodic motions) and hence influence its thinking.

- *Mathematics and physics: Students’ necessity to synthesize aspects of the two subjects; teachers’ suggestions.*

Mathematical and physics school practices are different cultural activities, since they have different goals, purposes, and objectives. In general, mmathematics aims to use problem solving to describe quantitative and spatial relationships of the physical environment (National Council of Teachers of Mathematics, 2000) while physics seeks through use of inquiry to describe and explain generalized patterns of events in the natural world (National Research Council, 1996). Besides, our analysis highlights *ontological differences* between physics and mathematics when ascending from observations to generalizations. Particular, mathematical evidence-based reasoning seems to be safer and more reliable than experimental evidence-based reasoning. The role of evidence — circumstantial or supportive — seems to be a main issue in scientific reasoning and the tentative nature of science (Ohertman & Lawson, 2008). Above and beyond their differences, we argue that mathematics and physics share: common practices (e.g., engage students in argumentation and reasoning); common teaching tools on periodicity (e.g., the sinusoidal function); and common reasoning behaviors when they argue on the validity of students’ justifications. Connecting mathematics and physics instruction is considered as a central issue in the contemporary research literature (Frykhlom & Glasson, 2005) since it can strengthen students’ understanding of common and neighbouring notions. Furthermore, we argue on the complexity of the engineering context where the above connections are not an option for teachers and students but are obligatory practices.

Our study provides evidence on the difficulties students face when there was a need to integrate knowledge from the two subjects and overcome conflicts among them (e.g., misconceptions about the function $f(x) = e^{-bx}\sin(x)$) or provide sufficient explanations on an engineering context task. This creates a necessity for teachers to help their students to develop a unified view on periodicity where aspects of the notion from the different subjects coexist harmonically. Some of teachers’ suggestions on the above necessity were: provide many examples of everyday life periodic phenomena and help students make links of the concrete situations with the mathematical objects that model them (e.g., by using animations of period motions). The above teachers’ suggestions are important but we notice that the practice of co-operation among colleagues of mathematics and science seems to be almost neglected by teachers. In this way, the student is ‘left alone’ to make the appropriate connections and overcome diversities among mathematics and physics common conceptions. To our view co-operation among teachers of mathematics and science could help students realize the differences and the commonalities mentioned above in the two subjects.

- *Productive links between students’ conceptualization and the production of sufficient justifications and explanations*

Argumentation and reasoning seems to be a non-familiar practice for undergraduate students. In the case of participating in this type of practice mostly logical-empirical types of justifications were identified in their responses while rarely did they use formal definitions as warrants in their responses (e.g., even mathematics undergraduate students avoid using the formal mathematical definition for periodic functions). One reason for students' reluctance to argue on their claims could be either because argumentative-pedagogy is not a common practice in the Greek educational system or because certain conceptual elements are not consciously comprehensible by them. We argue that students' explanations became more sophisticated when moving from partial to sufficient explanations. We identified that this differentiation depends on students' abilities to express connections between different fields of knowledge in an argumentative way. In this case, context-dependency is an action that fosters students in developing a more articulated, and thereby more elaborated understanding of the notion is occured.

Didactical implications

Findings can inform school textbooks' authors as they highlight the importance of the sequence of modes of reasoning and argumentation in students' conceptualization of periodical phenomena.

Teachers' awareness of textbooks' reasoning practices can play an important role in teaching interventions. Especially mathematics and science teachers need to discern the ontological and epistemological differences of science and mathematics textbooks in terms of periodicity in order to be able to fill in reasoning and conceptual gaps.

To our view, practices that could help students in this direction could be:

- students' participation in a wide range of reasoning practices (from sensory perceptions to abstract thinking and reasoning) on periodicity where they could overcome conflicts and conjectures;
- students' participation in contextual activities with real life periodic phenomena (e.g., problem solving);
- and teachers' familiarization with cooperation practices with their colleagues in neighboring subjects. This co-operation could involve sharing teaching experiences and familiarizing with each other perceptions about the notion.

Some of our suggestions are:

- Developing innovative classroom material linked with contemporary learning and teaching theories and that could initiate students' integration of the two cultures of inquiry (mathematical and scientific).
- Developing inquiry – based learning activities in our classrooms. Traditional educational systems have worked in a way that discourages the natural process of inquiry. Students learn not to ask too many questions, instead to listen and repeat the expected answers. In inquiry - based learning students are engageded in scientific reasoning and thinking; in producing alternative arguments; in examining hypotheses and conjectures; and in responding thoughtfully to their peers' and teachers' mathematical and scientific claims (Maaß & Artigue, 2013).
- Developing interdisciplinary communities of inquiry. The necessity for teachers in neighboring subjects to co-operate in their school is evident in our study. The teachers could share teaching experiences, provide teaching suggestions, reflect on their practices and improve them (Jaworski, 2006).

The above could contribute to students' understanding of a scientific notion that is an integral part of many scientific fields.

REFERENCES

Alexaki, N., Ampatzi, S., Gkougkousi, I., Kountouri, B., Mosxovete, N., Ovadia, S., Petroxeilo, K., Samprako, M., & Psalida, A. (2012). *Physics, 2nd year of upper secondary school, common core subject*. Athens, Greece: ITYE DIOFANTOS. [in Greek]

Anderson, C. W., & Smith, E. L. (1987). Teaching science. In V. Richardson-Koehler (Ed.), *Educators' handbook: A research perspective* (pp. 84-111). New York, NY: Longman.

Andreadakis, S., Katsargyris, B., Papastavridis, S., Polyzos, J., & Sverkos, A. (2012). *Algebra, 2nd year of upper secondary school, common core subject*. Athens, Greece: ITYE DIOFANTOS. [in Greek]

Andrinos, N. Panagiotidis, P. & Papadopoulos N. (2009). Car Systems I, Athens: OEDB, p. 235.

Arcavi, A. (2003). The role of visual representations in the teaching and learning of mathematics. *Educational Studies in Mathematics*, *52*(3), 215-241.

Biehler, R. (2005). Reconstructing of meaning as a didactical task: the concept of function as an example. In J. Kilpatrick (Ed.), *Meaning in Mathematics Education* (pp. 61-82). New York, NY: Springer.

Bills, L., Dreyfus, T., Mason, J., Tsamir, P., Watson, A., & Zaslavsky, O. (2006). Exemplification in mathematics education. In J. Novotná, H. Moraová, M. Krátká, & N. Stehliková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 126-154). Prague, Czech Republic: PME.

Bingolbali, E., & Monaghan, J. (2008). Concept image revisited. *Educational Studies in Mathematics*, *68*(1), 19-35.

Bliss, J., Monk, M., & Ogborn, J. (1983). *Qualitative data analysis for educational research*. London, England: Croom Helm.

Braaten M, Windschitl M. (2011). Working toward a stronger conceptualization of scientific explanation for science education. *Science Education*, 95(4), 639–669.

Buendia G. & Cordero F. (2005). Prediction and the periodical aspect as generators of knowledge in a social practice framework. *Educational Studies in Mathematics 58*(3), 299– 333.

Chi, M. T. H., de Leeuw, N., Chiu, M., & LaVancher, C. (1994). Eliciting self-explanations improves understanding. *Cognitive Science,* 18*,* 439-477.

Clark, R.C., & Mayer, R.E. (2008). Learning by viewing versus learning by doing: Evidence-based guidelines for principled learning environments. *Performance Improvement,* 47, 5–13.

Corbin, J., & Strauss, A. (2007). Basics of qualitative research: Techniques and procedures for developing grounded theory (3rd ed.). Thousand Oaks, CA: Sage.

Dolev, S., & Even, R. (2013). Justifications and explanations in Israeli 7th grade math textbooks. *International Journal of Science and Mathematics Education.* Advance online publication. doi:10.1007/s10763-013-9488-7

Douek, N. (1999b). Argumentation and conceptualisation in context: a case study on sun shadows in primary school. *Educational Studies in Mathematics*, *39*, 89–110.

Dreyfus, T. & Eisenberg, T. (1980). On teaching periodicity. *International Journal in Mathematics Education Science & Technology*, *11*(4), 507-509.

Fahnestock, J., & Secor, M. (1988). The stases in scientific and literary argument. *Written Communication*, *5*, 427–443.

Font, V., Godino, J. D. & D'Amore, B. (2007) An onto-semiotic approach to representations in mathematics education. *For the Learning of Mathematics*, *27* (2), 2–7.

Frykholm, J. & Glasson, G. (2005). Connecting Science and Mathematics Instruction: Pedagogical Content Knowledge for Teachers. *School Science and Mathematics*. *105*(3), 127-141.

Gasson, S. (2003). Rigor in grounded theory research - An interpretive perspective on generating theory from qualitative field studies. In M. Whitman & A. Woszczynski (Eds.), *Handbook for information systems research* (pp. 79-102). Hershey, PA: Idea Group.

Groarke, L. (1996). Logic, art and argument. *Informal Logic, 18,* 105-129.

Haggarty, L. and Pepin, B. (2002) 'An investigation of mathematics textbooks and their use in English, French and German Classrooms: who gets an opportunity to learn what?' *British Educational Research Journal* **28** (4).

Halliday, M.A.K. (1985). *An Introduction to Functional Grammar.* London: Edward Arnold.

Herbel-Eisenmann, B. & Wagner, D. (2007). How a textbook might position a mathematics learner. *For the Learning of Mathematics*, *27*(2), 8-14.

Hsu, P., & Yang, W. (2007). Print and image integration of science texts and reading comprehension: A systemic functional linguistics perspective. *International Journal of Science and Mathematics Education, 5*(4), 639-659.

Jaworski, B. (2006). Theory and practice in mathematics teaching development: critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, 9(2), 187-211.

Jiménez-Aleixandre, M., & Pereiro-Muñoz, C. (2002). Knowledge producers or knowledge consumers? Argument and decision making about environmental management. *International Journal of Science Education*, 24(11), 1171-1190.

Koponen, I. T., & Nousiainen, M. (2012). Pre-service physics teachers' understanding of the relational structure of physics concepts: Organising subject content for purposes of teaching. *International Journal of Science and Mathematics Education*, *11*(2), 325-357.

Kozma, R.B., Chin, E., Russell, J., & Marx, N. (2000). The roles of representations and tools in the chemistry laboratory and their implications for chemistry instruction. *Journal of the Learning Sciences*, 9, 105–143.

Kress, G & van Leeuwen, T. (2006). *Reading Images, the Grammar of Visual Design* (2nd ed.). London: Routledge.

Kublikowski, R. (2009). Definition within the structure of argumentation. *Studies in Logic, Grammar and Rhetoric, 16*(29), 229-244**.**

Lakatos, I. (1976). *Proofs and refutations.* Cambridge, England: Cambridge University Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29-64.

Latour, B. (1987). *Science in action: How to follow scientists and engineers through society*. Milton Keynes: Open University Press.

Lawson, A. E. (2004). The nature and development of scientific reasoning: A synthetic view. *International Journal of Science and Mathematics Education*, *2,* 307–338.

Lektorsky, V. A. (1995). Knowledge and cultural objects. In L. Kuçuradi & R. S. Cohen (Eds.), The concept of knowledge. The Anakara seminar (pp. 191–196). Dordrecht: Kluwer.

Levin, J. R. (1981). On functions of pictures in prose. In Pirozzolo, F. J., and Wittrock, M. C. (eds.), *Neuropsychological and Cognitive Processes in Reading*, Academic Press, New York, pp. 203–228.

Lin, F. L., & Yang, K. L. (2007). The reading comprehension of geometric proofs: The contribution of knowledge and reasoning. *International Journal of Science and Mathematics Education, 5*(4), 729-754.

Love, E., & Pimm, D. (1996). 'This is so': A text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International Handbook of Mathematics Education* (Vol. 1, pp. 371-409). Dordrecht: Kluwer Academic Publishers

Maaß, K. & Artigue, M. (2013). Implementation of inquiry-based learning in day-to-day teaching: a synthesis, *ZDM*, 45(6), pp 779-795.

Mahidi, S. (2013). A comparison between the knowledge organization of university physics teachers and the textbooks they use for their teaching purposes: Biot-Savart Law and Ampère's Law. *International Journal of Science and Mathematics Education*, http://link.springer.com/article/10.1007/s10763-013-9457-1.

Mayring, P. (2000). Qualitative Content Analysis [28 paragraphs]. Forum Qualitative Sozialforschung / Forum: *Qualitative Social Research*, 1(2), Art. 20, http://nbn-resolving.de/urn:nbn:de:0114-fqs0002204.

McComas, W. F. (2003). A textbook case of the nature of science: Laws and theories in the science of biology. *International Journal of Science and Mathematics Education*, *1*(2), 141–155.

Mesa, V. (2004). Characterizing practices associated with functions in middle school textbooks: an empirical approach. *Educational Studies in Mathematics,56*, 255-286.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Research Council. (1996). *National science education standards.* Washington, DC: National Academies Press.

Nicol, C. & Crespo, S. (2006). Learning to teach with textbooks: How pre-service teachers interpret and use curriculum materials in their teaching. *Educational Studies in Mathematics: An International Journal, 62*(3). Kluwer, The Netherlands.

Norris, S. P., & Phillips, L. M. (2003). How literacy in its fundamental sense is central to scientific literacy. *Science Education, 87*(2), 224–40.

Nunez, R. (2007). The cognitive science of mathematics: Why is it relevant for mathematics education? In R. A. Lesh, E. Hamilton & J. Kaput (Eds.), *Foundations for the future in mathematics education*. London: Lawrence Erlbaum Associates.

Oehrtman, M., & Lawson, A. E. (2008). Connecting science and mathematics: The nature of proof and disproof in science and mathematics. *International Journal of Science and Mathematics Education, 6,* 377- 403.

Pannekoek, A. (1961). *A history of astronomy*. New York: Dover.

Pea, R. D. (1993). Practices of distributed intelligence and designs for education. In G. Salomon (Ed.), Distributed cognitions (pp. 47–87). Cambridge: Cambridge University Press.

Pegg, J., & Karuku, S. (2012). Explanatory reasoning in junior high science textbooks. In S. P. Norris (Ed.), *Reading for evidence and interpreting visualizations in mathematics and science education* (pp. 65-81). Rotterdam, The Netherlands: Sense.

Perkins DN., Grotzer TA. (2005). Dimensions of causal understanding: The role of complex causal models in students' understanding of science. *Studies in Science Education*, 41, 117-166.

Piaget, J. (1928). *Judgment and reasoning in the child*. London: Paul Kegan.

Pozzer-Adernghi, L. & Roth, W.M. (2004). Making sense of photographs. *Science Education, 89*, 219-241.

Radford, L. (2013). On the role of representations and artefacts in knowing and learning, Educational studies in mathematics, DOI 10.1007/s10649-013-9527-x.

Radford, L., Cerulli, M., Demers, S., & Guzmán, J. (2004). The sensual and the conceptual: Artefact mediated kinesthetic actions and semiotic activity. In M. J. Høines, & A. B. Fuglestad (Eds.), *Proceedings of the 28 Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 73–80). Norway: Bergen University College.

Rezat, S. (2006). The structures of German mathematics textbooks. *ZDM Mathematics Education*, *38*(6), 482-487.

Roth, W., & McGinn, M.K. (1997). Graphing: Cognitive ability or practice? *Science Education*, 81, 91–106.

Roth, W.M. & Lee, Y.J. (2007). “Vygotsky’s Neglected Legacy”: Cultural-Historical Activity Theory. *Review of Educational Research*, 77, 186-232.

Sfard, A. & McClain, K. (2002). Analyzing Tools: Perspectives on the Role of Designed Artifacts in Mathematics Learning, *The Journal of the Learning Sciences*, 11(2&3), 153–161.

Shama, G. (1998). Understanding periodicity as a process with a gestalt structure. *Educational Studies in Mathematics*, 35, 255-281.

Stacey, K., & Vincent, J. (2009). Modes of reasoning in explanations in Australian eighth-grade mathematics textbooks. *Educational Studies in Mathematics*, *72*(3), 271-288.

Stinner, A. (1992). Science textbooks and science teaching: From logic to evidence. *Science Education*, *76*(1), 1-16.

Strauss, A. L., & Corbin, J. (2007). *Basics of qualitative research: Techniques and procedures for developing grounded theory, 3rd ed*. SAGE Publications.

Stylianides, G. J. (2008). Investigating the guidance offered to teachers in curriculum materials: The case of proof in mathematics. *International Journal of Science and Mathematics Education*, *6*(1), 191-215.

Stylianides, G. J. (2009). Reasoning–and-proving in school mathematics textbooks. *Mathematical Thinking and Learning*, *11*(4), 258-288.

Tabach, M., Levenson, E., Barkai, R., Tirosh, D. Tsamir, P., & Dreyfus, T. (2010). Secondary school teachers' awareness of numerical examples as proof. Research in Mathematics Education, 12(2), 117-131.

Toulmin, S. (2003). *The uses of argument* (updated ed.). Cambridge, England: Cambridge University Press.

Triantafillou, C., Spiliotopoulou, V. & Potari, D. (2014). How undergraduate students make sense out of graphs: the case of periodic motions. 2014. In Nicol, C., Oesterle, S., Liljedahl, P., & Allan, D. (Eds.) *Proceedings of the Joint Meeting of PME 38 and PME-NA 36,* Vol. 5, pp. 273-280. Vancouver, Canada.

Triantafillou, C., Spiliotopoulou, V. (2014). Visual Representations of the Concept of Periodicity across Subjects: Their Context and Genre. In C. P. Constantinou, N. Papadouris & A. Hadjigeorgiou (Eds.), *E-Book Proceedings of the ESERA 2013 Conference: Science Education Research For Evidence-based Teaching and Coherence in Learning*. Part 2, (pp. 334-344) Nicosia, Cyprus: European Science Education Research Association. ISBN: 978-9963-700-77-6.

Triantafillou, C., Spiliotopoulou, V., & Potari, D. (2015). The Nature of Argumentation in School Mathematics and Physics Texts: The Case of Periodicity. *International Journal in Mathematics and Science Education*, DOI 10.1007/s10763-014-9609-y.

van Dormolen, J. & Zaslavsky, O. (2003). The many facets of a definition: the case of periodicity. *Journal of Mathematical Behavior*, 22(1), 1-106.

van Dormolen, J. (1986). Textual analysis. In B. Christiansen, A. G. Howson, & M. Otte (Eds.), *Perspectives on mathematics education* (pp. 141-171). Dordrecht, The Netherlands: D. Reidel.

van Eemeren, F.H. & Grootendorst, R. (2004). *A Systematic Theory of Argumentation: The Pragma-dialectical Approach*. Cambridge UK: Cambridge University Press.

Venville, G. J & Dawson, V. M. (2010). The impact of a classroom intervention on grade 10 students' argumentation skills, informal reasoning, and conceptual understanding of science. *Journal of Research in Science Teaching*, 47 (8), 952–977.

Vygotsky, L. S. (1978). *Mind in society: the development of higher psychological processes*. Cambridge: Harvard University.

Whitehead, A. N. (1983). *Concept of Nature*. Cambridge: Cambridge University Press.